ECE 105: Introduction to Electrical Engineering

Lecture 3

Circuit analysis 1

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Basic circuit quantities

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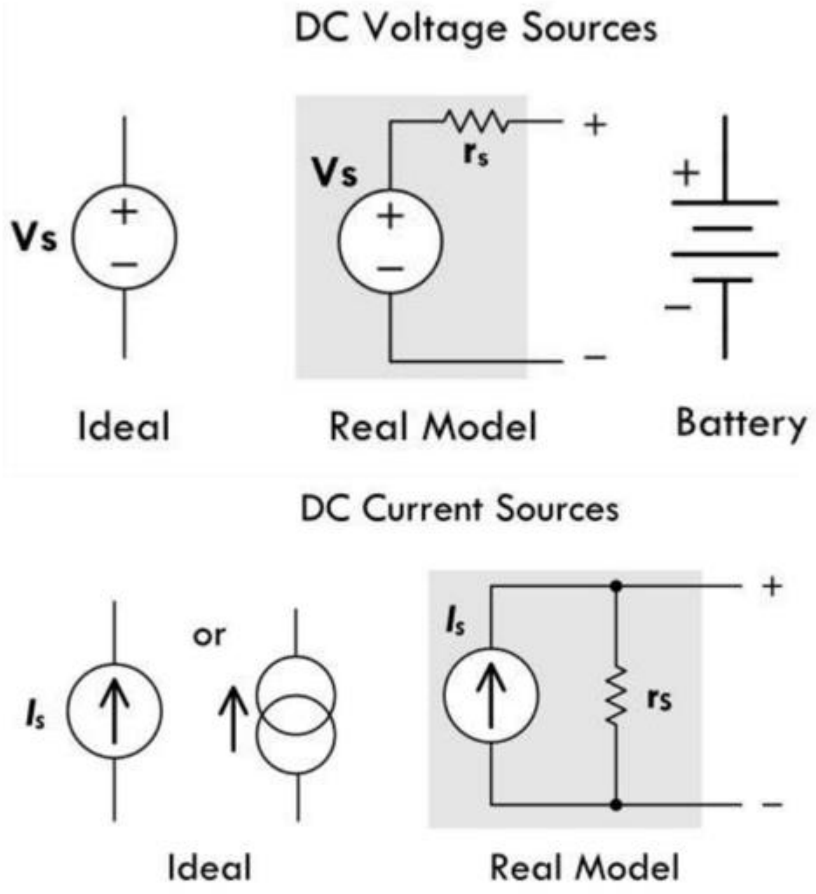
Quantity	Symbol	Units
Voltage	V	Volts (V)
Current	I	Amperes (A)
Resistance	R	Ohms (Ω)
Capacitance	C	Coulomb (C)
Inductance	L	Henry (H)

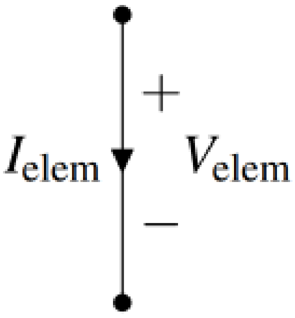
Notations



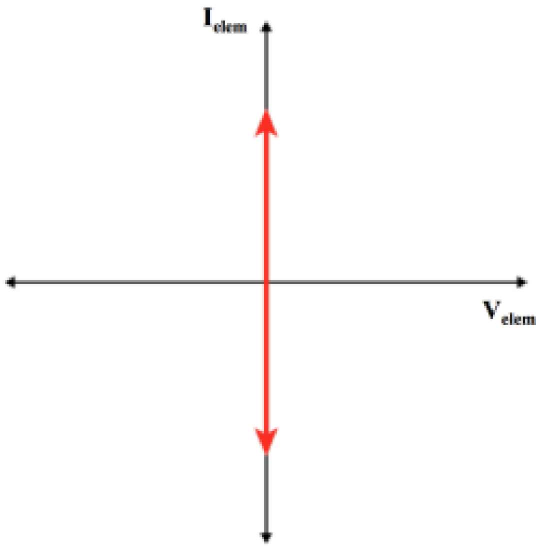
DC Voltage Sources
Vs
+
–
Vs
+
–
r_s
+
–
+
–
Ideal
Real Model
Battery
DC Current Sources
I_s
or
I_s
r_s
+
–
Ideal
Real Model

Wire

Symbol



IV Relationship

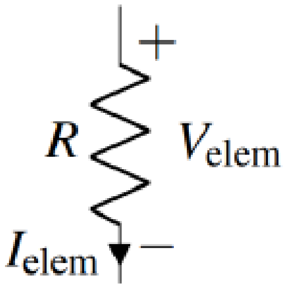


Wire: The most common element in a schematic is the wire, drawn as a solid line. The IV relationship for a wire is:

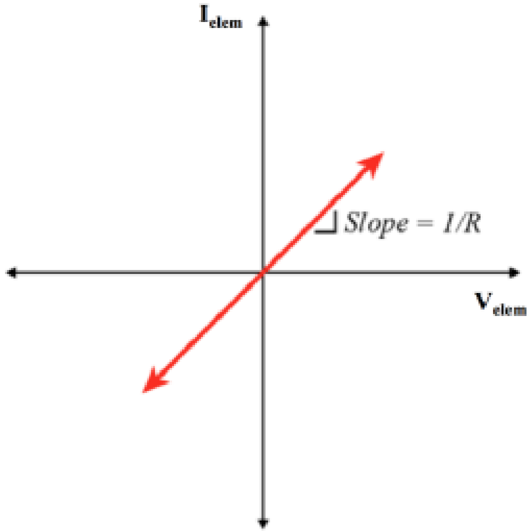
- $V_{\text{elem}} = 0$
  A wire is an ideal connection with zero voltage across it.
- $I_{\text{elem}} = ?$
  The current through a wire can take any value, and is determined by the rest of the circuit.

Resistor

Symbol



IV Relationship



Resistor: The IV relationship of a resistor is called "Ohm's Law."

- $V_{\text{elem}} = I_{\textbf{elem}} \times R$
  The voltage across a resistor is determined by Ohm's Law.
- $I_{\text{elem}} = \frac{V_{\text{elem}}}{R}$
  The current through a resistor is determined by Ohm's Law.

Open circuit

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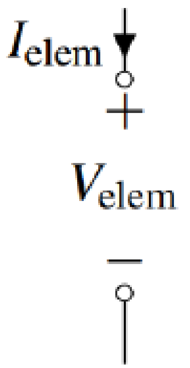
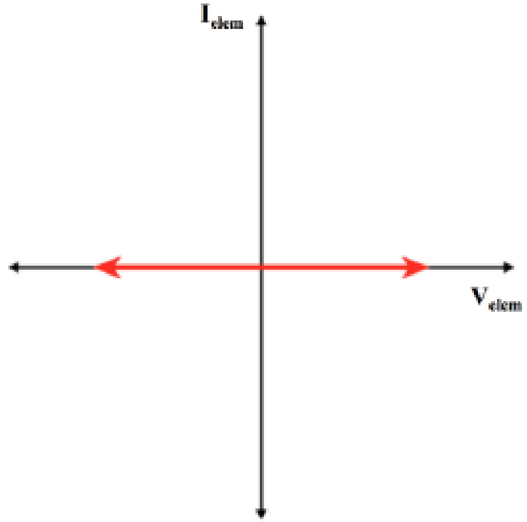
Symbol



IV Relationship

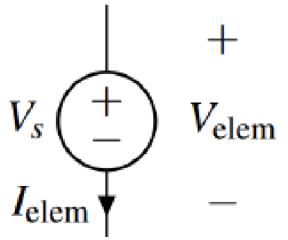


Open Circuit: This element is the dual of the wire.

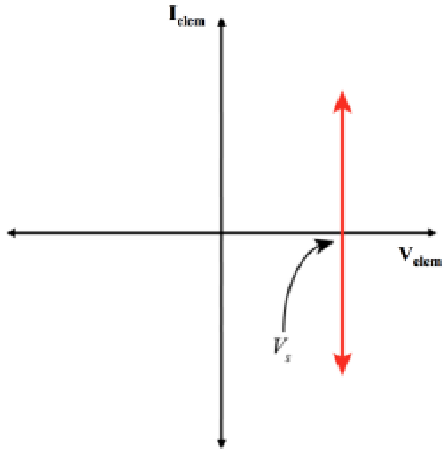
- $V_{elem} = ?$
  The voltage across an open circuit can take any value, and is determined by the rest of the circuit.
- $I_{elem} = 0$
  No current is allowed to flow through an open circuit.

Voltage source

Symbol



IV Relationship

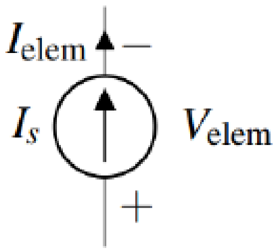


Voltage Source: A voltage source is a component that forces a specific voltage across its terminals. The + and − signs indicate which direction the voltage is oriented. The voltage difference between the + terminal and the − terminal is always equal to V_s, no matter what else is happening in the circuit.

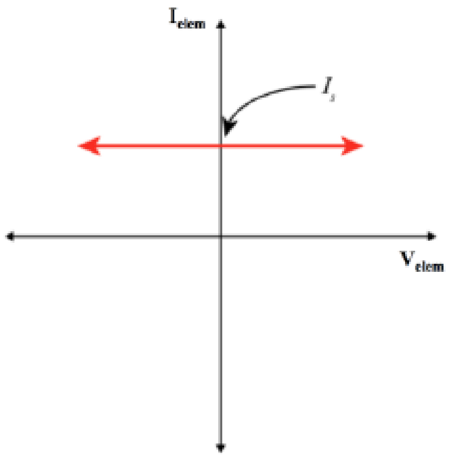
- $V_{\text{elem}} = V_s$
  The voltage across the voltage source is always equal to the source value.
- $I_{\text{elem}} = ?$
  The current through a voltage source is determined by the rest of the circuit.

Current source

Symbol



IV Relationship



Current Source: A current source forces current in the direction specified by the arrow indicated on the schematic symbol. The current flowing through a current source is always equal to I_S, no matter what else is happening in the circuit. Note the duality between this element and the voltage source.

- $V_{elem} =?$
  The voltage across a current source is determined by the rest of the circuit.
- $I_{elem} = I_S$
  The current through a current source is always equal to the source value.

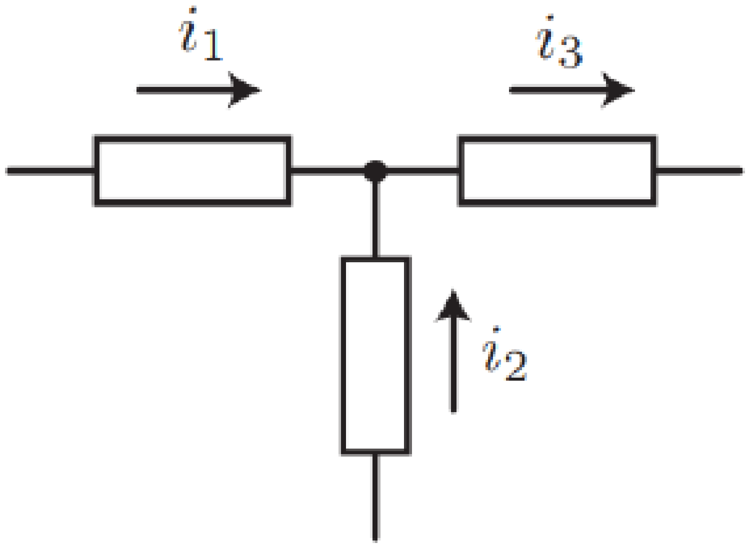
Rule of circuit analysis - KCL

Kirchhoff's Current Law (KCL)
A place in a circuit where two or more circuit elements meet is called a **node**. Kirchhoff's Current Law (KCL) states that the net current flowing out of (or equivalently, into) any node of a circuit is zero. To put this more simply, the current flowing into a node must equal the current flowing out of that node.
Mathematically, KCL states that:

$$\sum_{\text{Node}} i_k = 0. \tag{1}$$

For example, consider the circuit shown above. We define current flowing out of the node to be positive, and therefore, current flowing into the node is negative. From the left branch, there is i_1 current flowing into the node, so there is $-i_1$ current flowing out of the node. Similarly, there is $-i_2$ current flowing out of the node and i_3 current flowing out of the node. Therefore, the currents must satisfy:

$$(-i_1) + (-i_2) + i_3 = 0 \quad \text{or} \quad i_1 + i_2 = i_3. \tag{2}$$



Rule of circuit analysis - KVL

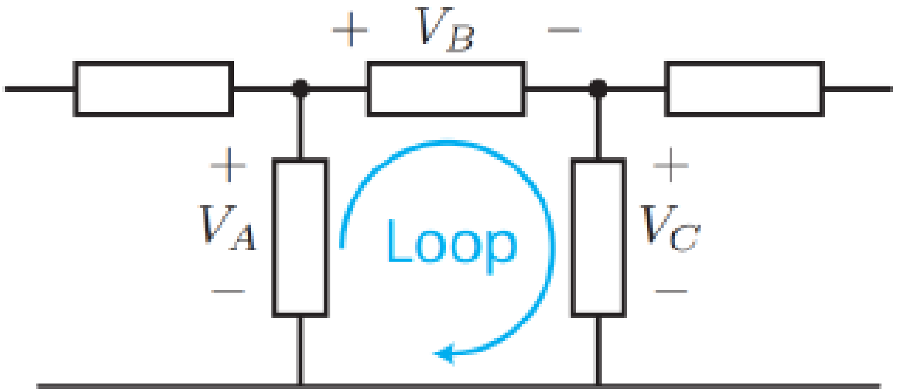
Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law (KVL) states that the sum of voltages across the elements connected in a loop must be equal to zero. In our elevation analogy for voltage, this is equivalent to saying "what goes up must come down".
Mathematically, KVL states that:

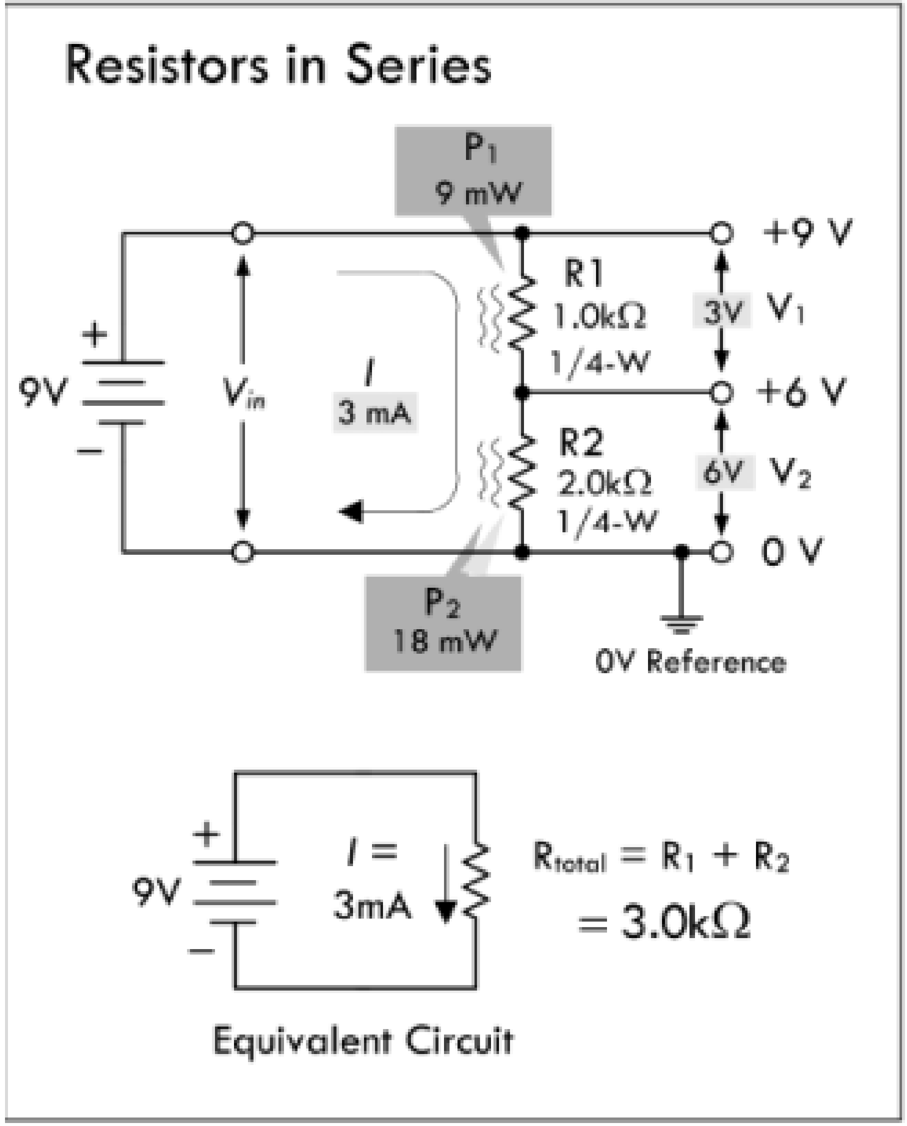
$$\sum_{\text{Loop}} V_k = 0. \tag{3}$$

When adding the voltage "drops" around the loop, we must follow a convention. If the arrow corresponding to the loop goes into the "+" of an element, we subtract the voltage across that element. (In our elevation analogy, we went "downhill" from higher voltage to lower voltage, so we lost "elevation.") Conversely, if the arrow goes into the "-" of an element, we add the voltage across that element (this is like going "uphill"). Following this convention for the example in Figure 11, we find:

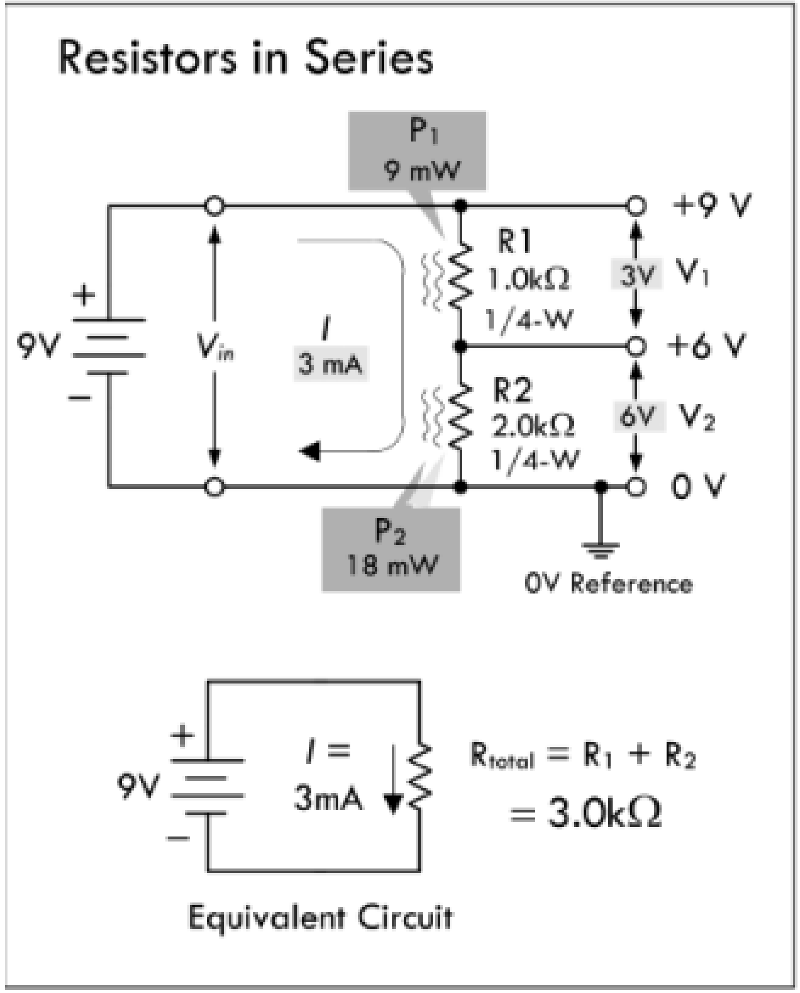
$$V_A - V_B - V_C = 0. \tag{4}$$

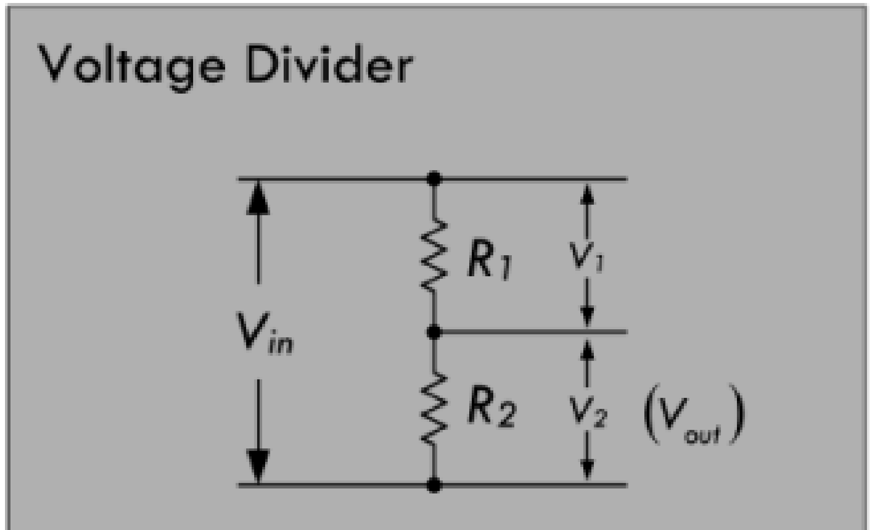


Resistors in series



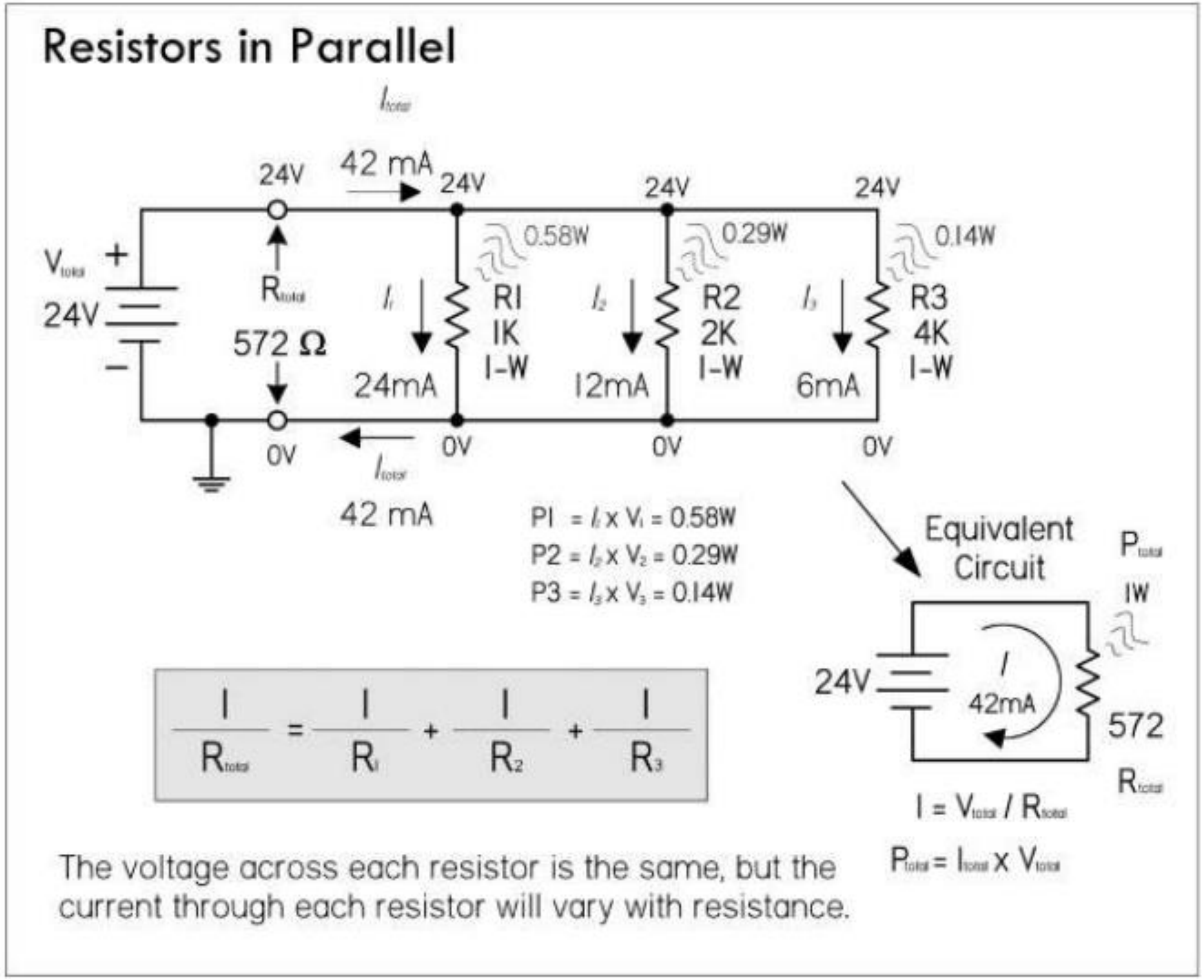
Voltage divider





$$V_1 = V_{in} \frac{R_1}{R_1 + R_2} \qquad V_2 = V_{in} \frac{R_2}{R_1 + R_2}$$

Resistors in parallel



Current divider

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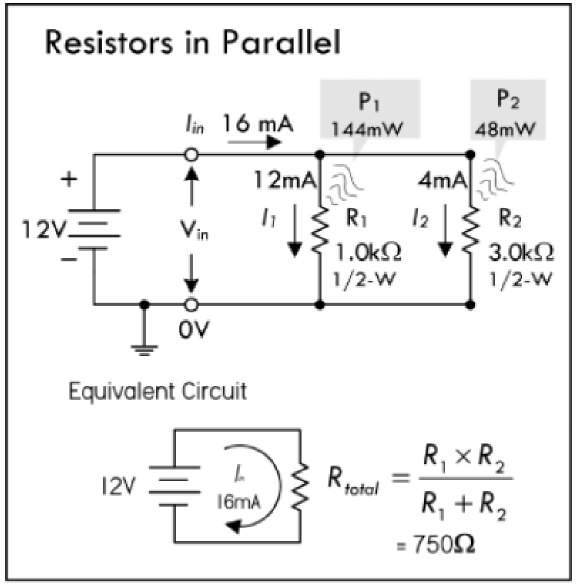
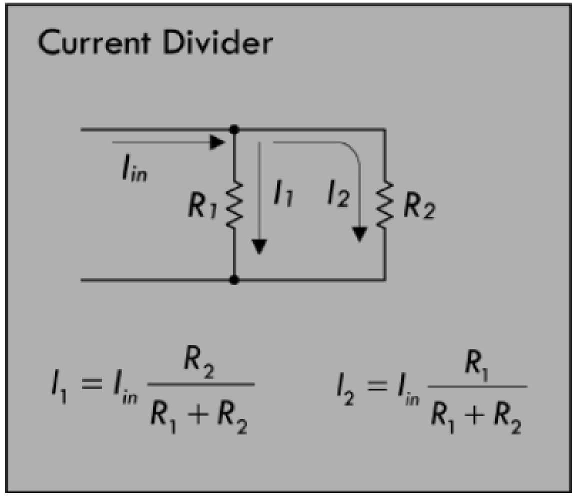
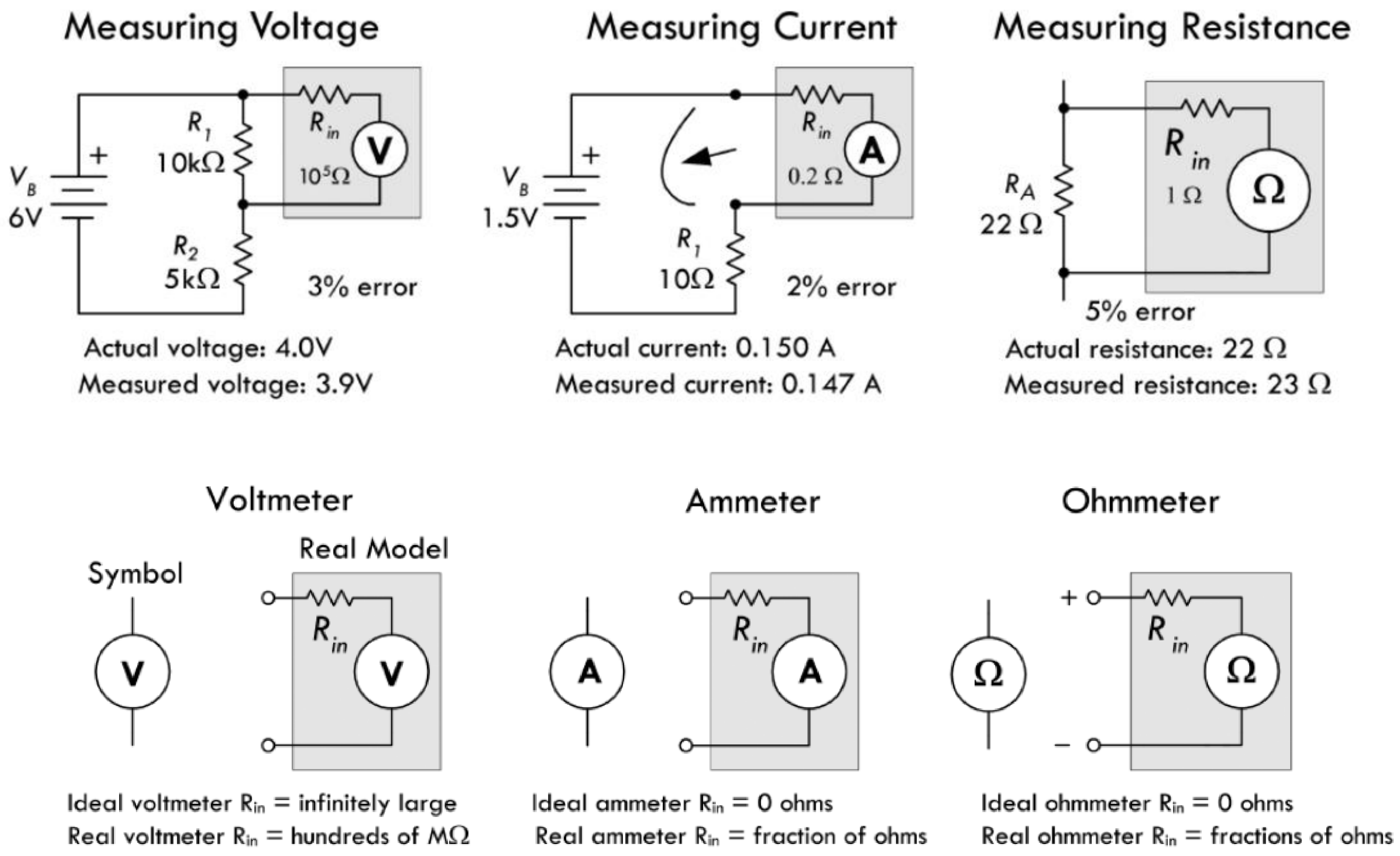
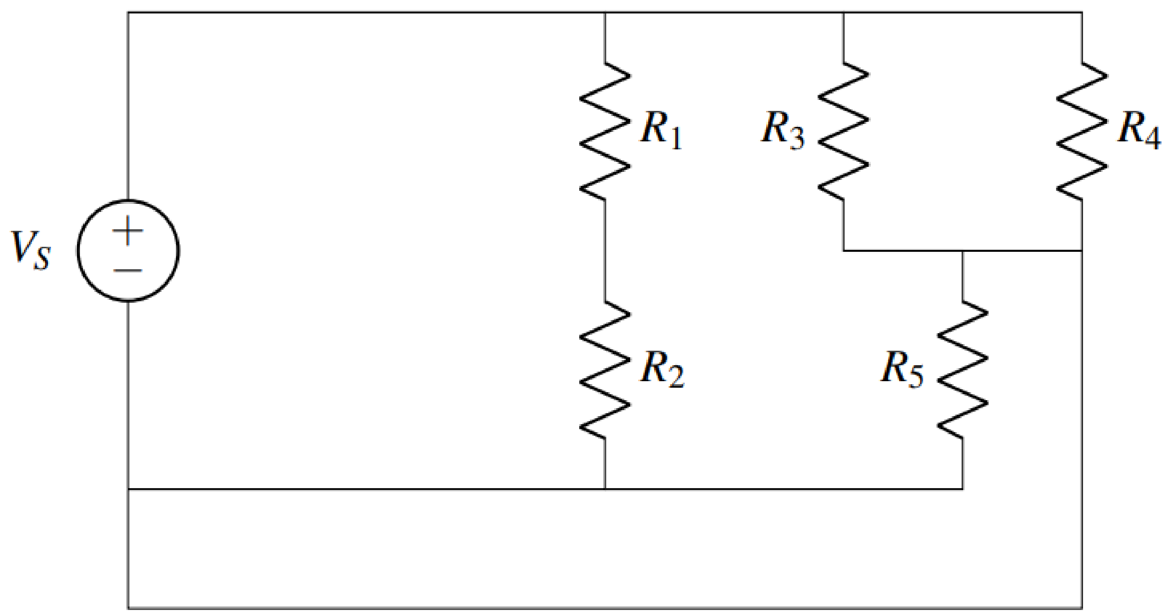




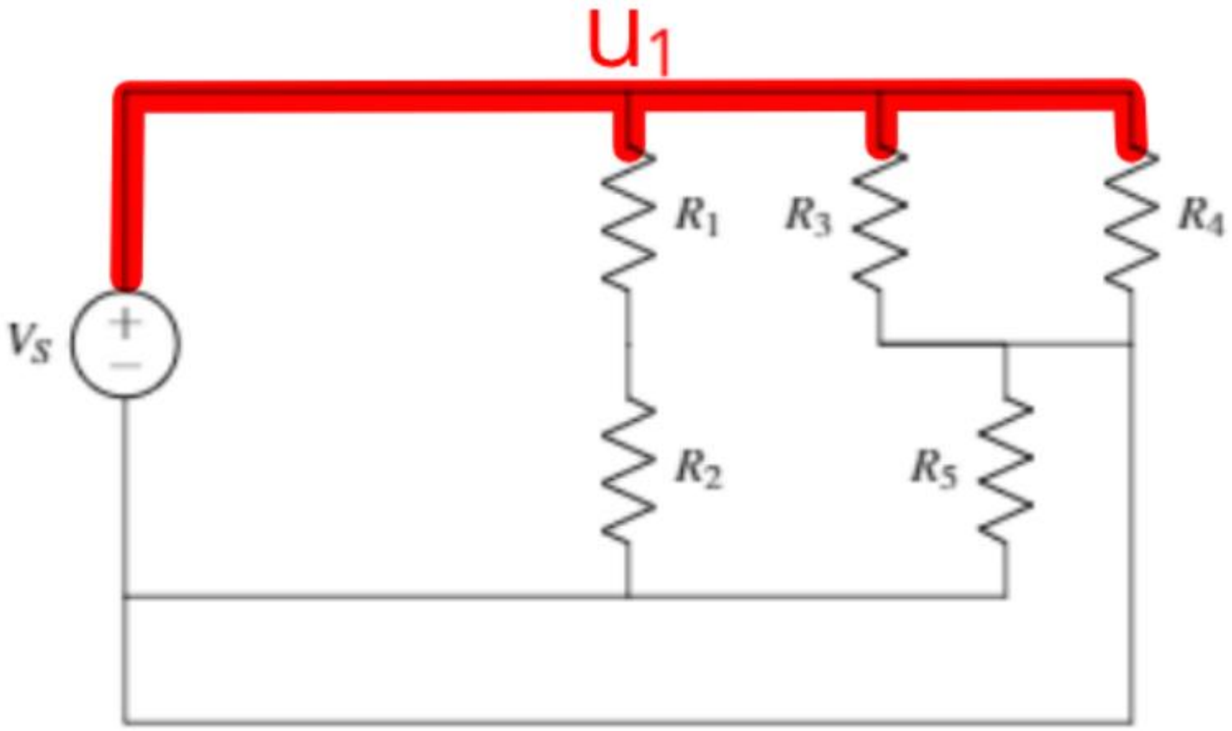
Measuring voltage, current, resistance



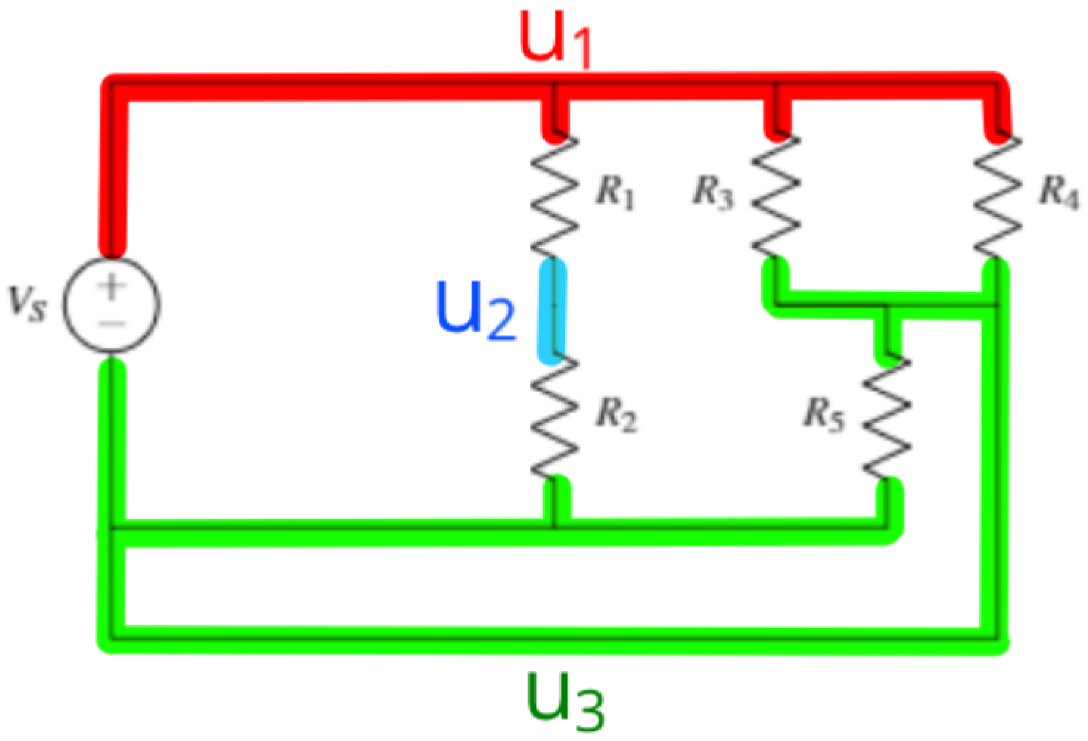
Finding nodes



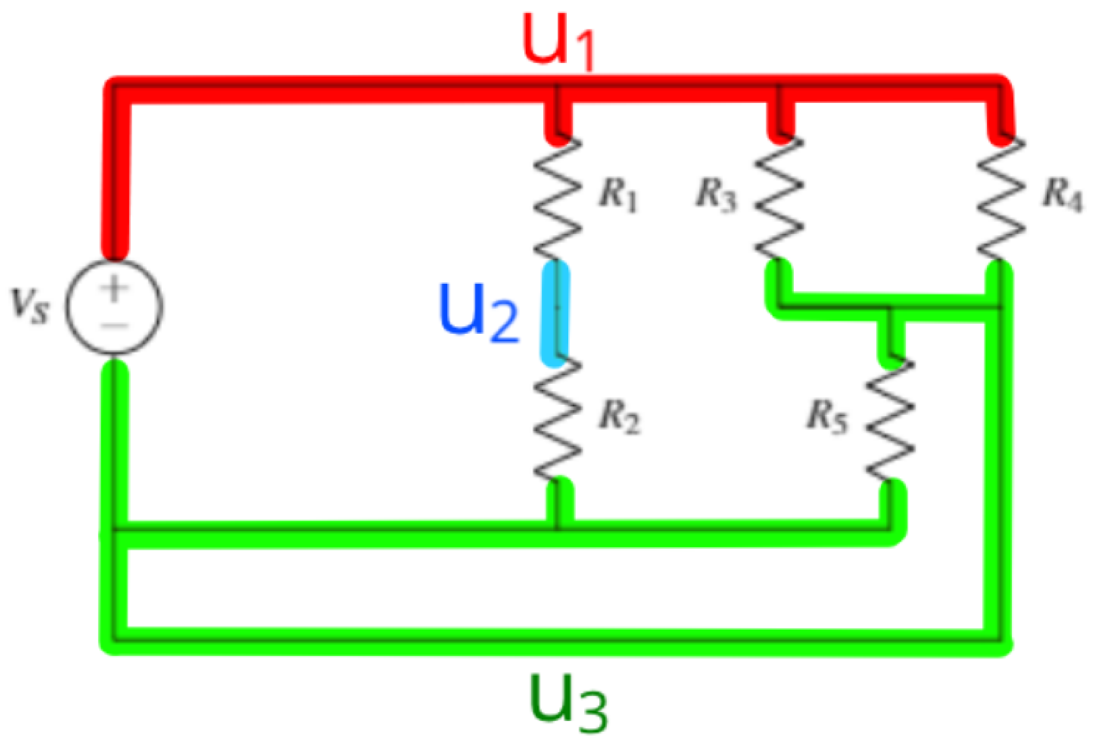
Find the node connected to the source

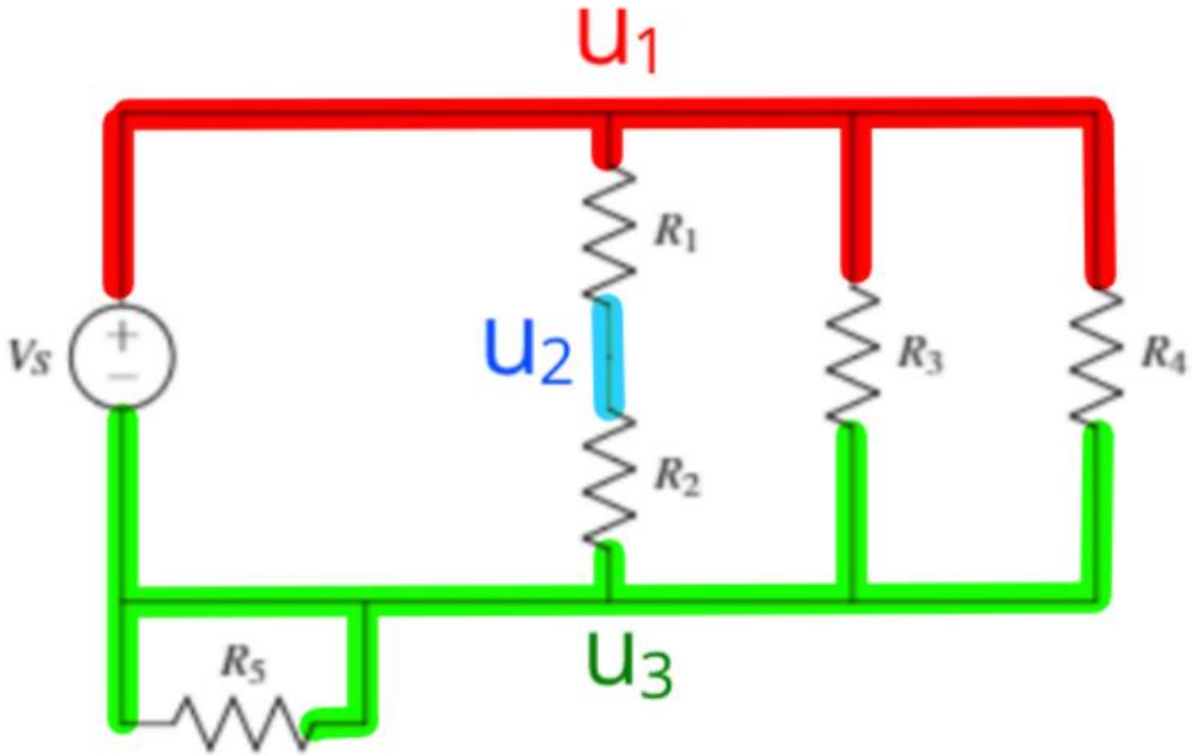


Find other nodes



Merge elements sharing the same nodes





Practice problems

1. True or False: A voltage source can have any current through it.
2. True or False: A current source can have any voltage across it.
3. True or False: The voltage across R1 and across R2 is the same
4. True or False: The current through the resistors is the same.

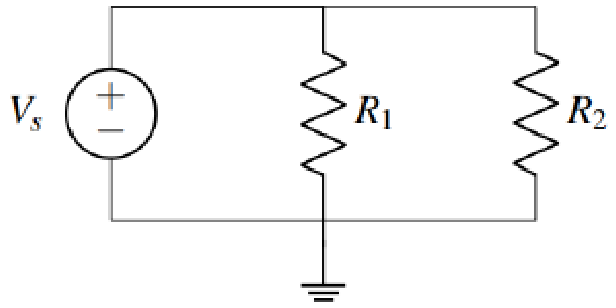


Figure 18: Question 3

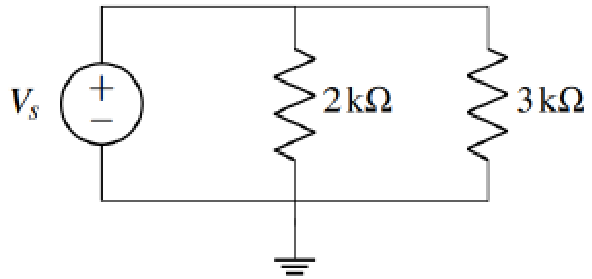


Figure 19: Question 4

Practice problems

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7. How many nodes are in the following circuit?

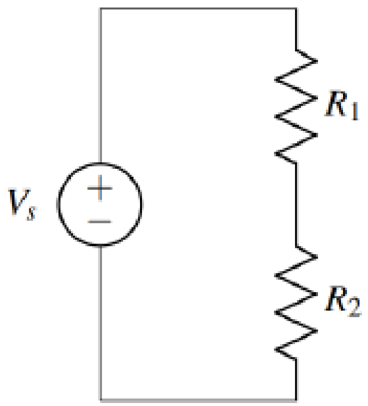
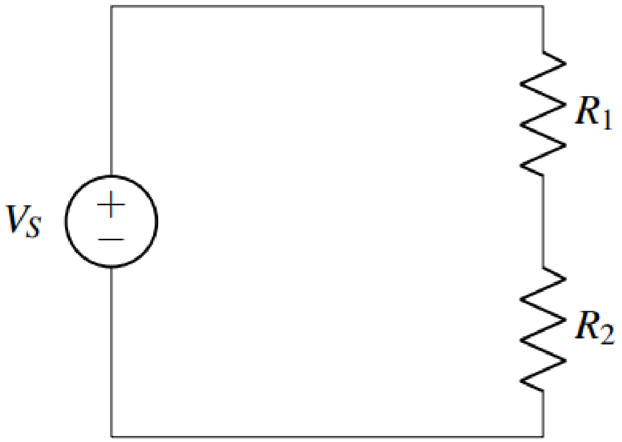
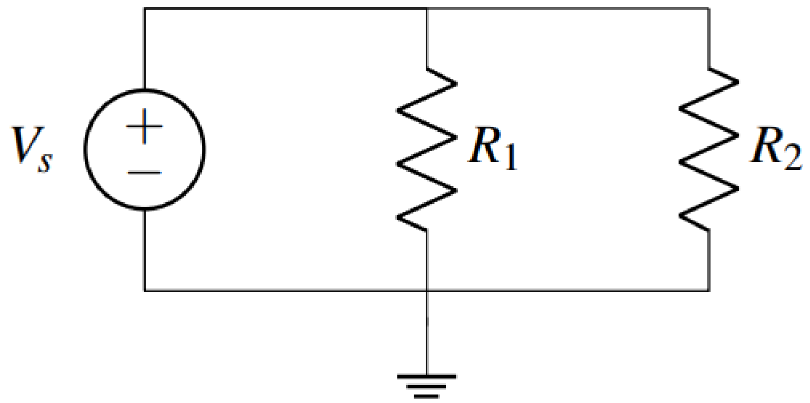


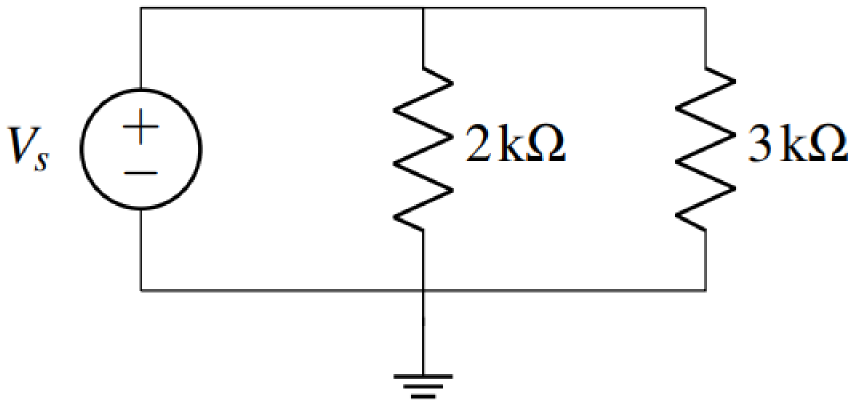
Figure 21: Question 7

Practice

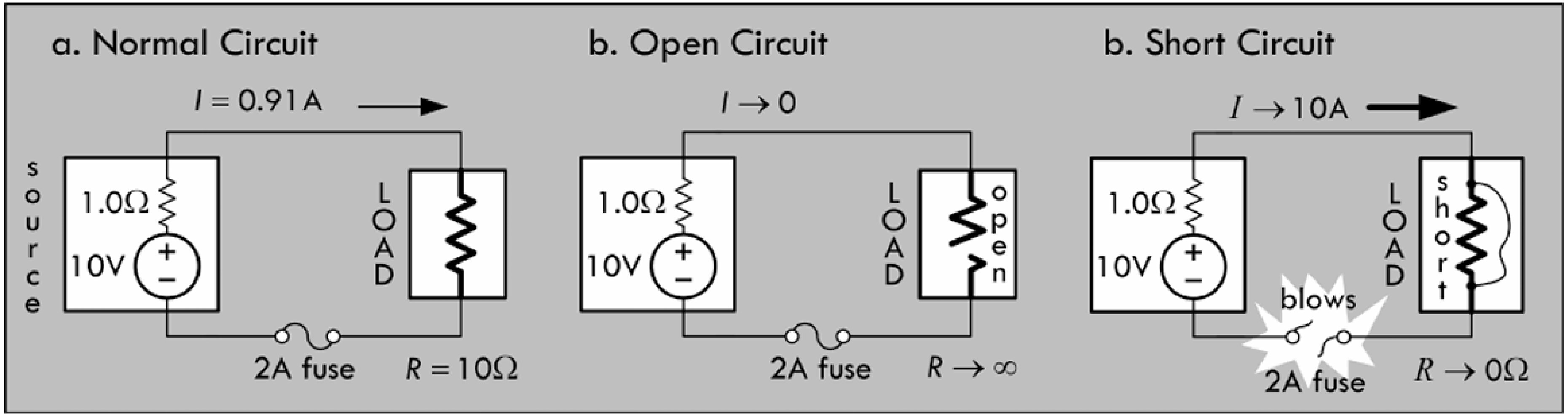


Practice

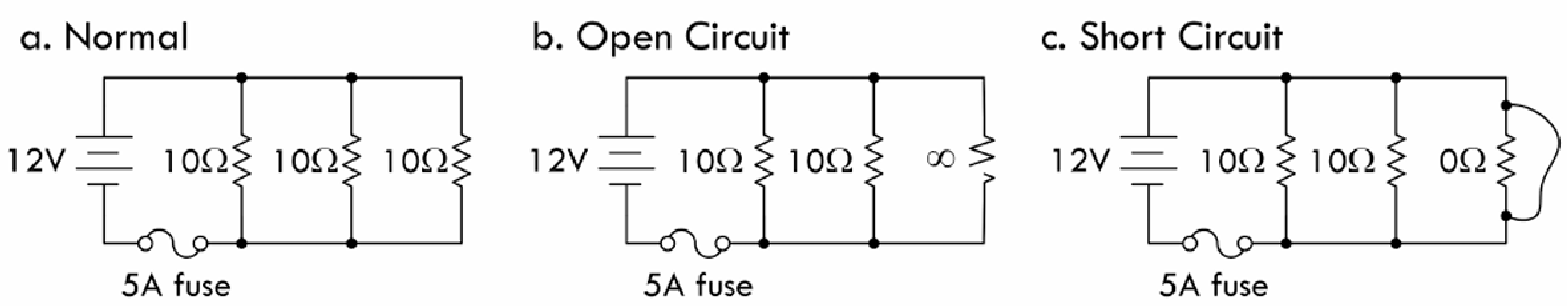




Fuse



a. Normal Circuit
I = 0.91 A
source
1.0Ω
10V
LOAD
2A fuse
R = 10Ω
b. Open Circuit
I → 0
1.0Ω
10V
LOAD
open
2A fuse
R → ∞
b. Short Circuit
I → 10A
1.0Ω
10V
LOAD
short
blows
2A fuse
R → 0Ω



a. Normal
12V
10Ω
10Ω
10Ω
5A fuse
b. Open Circuit
12V
10Ω
10Ω
∞
5A fuse
c. Short Circuit
12V
10Ω
10Ω
0Ω
5A fuse