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ECE 105: Introduction to Electrical Engineering

Lecture 4

Circuit Analysis 2

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Voltage across a current source

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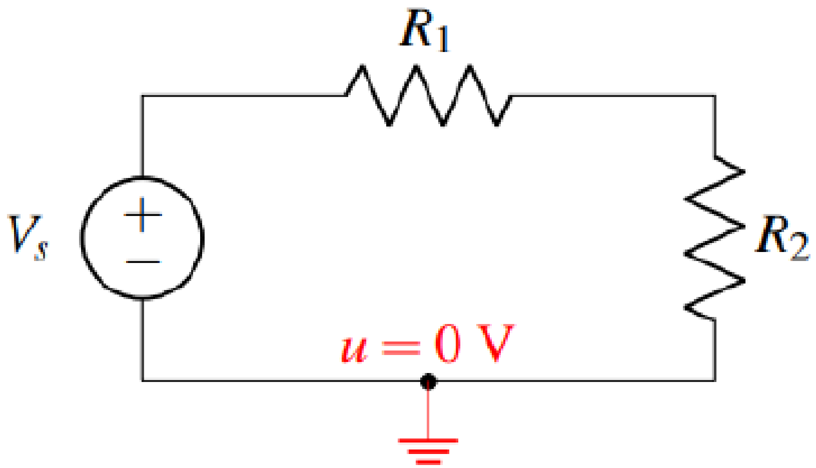
2. True or False: A current source can have any voltage across it.

- The voltage across an ideal current source is completely determined by the circuit it is connected to.

Nodal analysis

Step 1: Pick a node and label it as u = 0 V ("reference"), meaning that we will measure all of the other node voltages in the circuit relative to this point. Any node can be the reference node, but conventionally its the negative terminal of a voltage source. On the schematic we can also use a **ground symbol** to denote the reference node.



Label nodes

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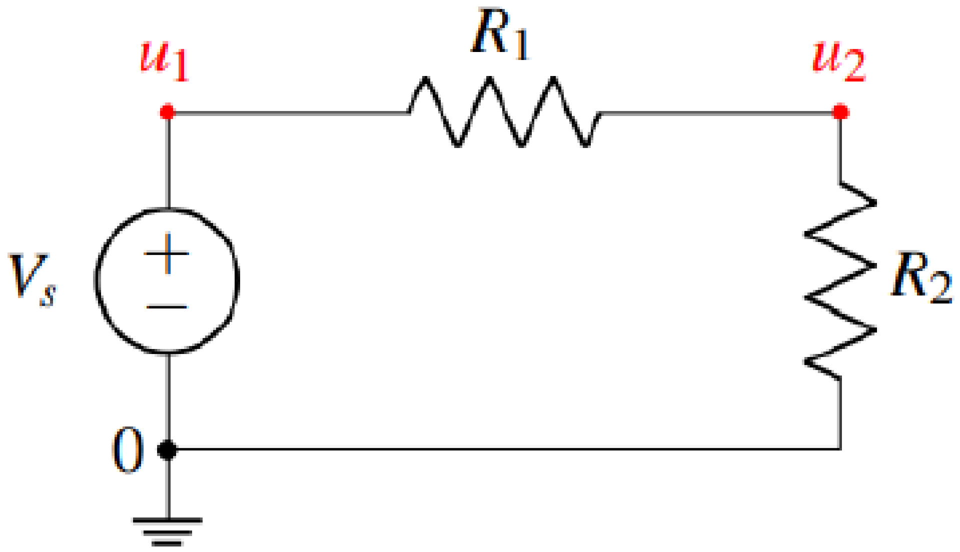
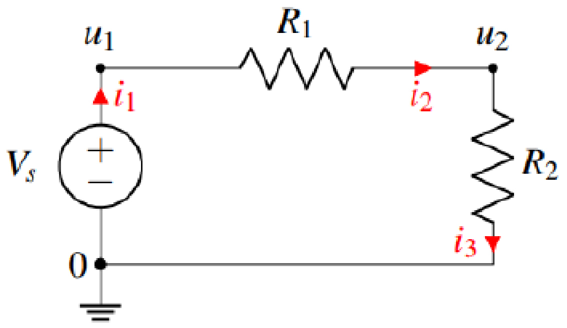
Step 2: Label all remaining nodes as some "ui", representing the voltage at each node relative to the zero/reference node.



Label currents

Step 3: Label the current through every element in the circuit "in" so that every element has a current label. The direction of the arrow indicates which direction of current flow you are considering to be positive. At this stage of the algorithm, you can pick the direction

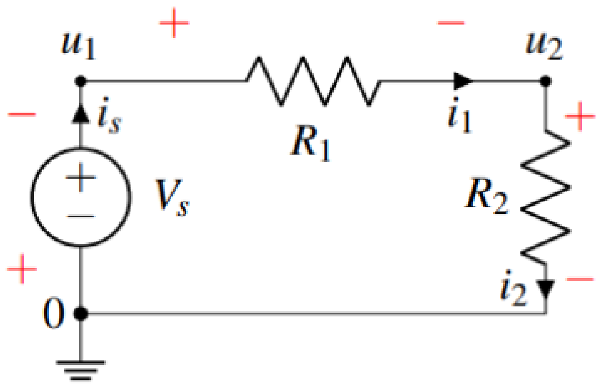
of all the current arrows arbitrarily. As long as you are consistent with this choice and follow the rules described in the rest of this algorithm, the math will work out correctly.



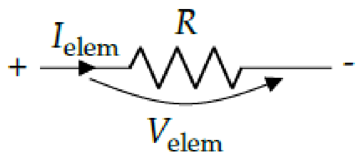
Note that we only label the current once for each element—for example, we can label i3 as the current leaving the resistor (as is done in the diagram) *or* we can label it as the the current entering the resistor. These are equivalent because KCL also holds within the element itself—i.e., the current entering an element must be equal to the current exiting that same element.

Voltage drop directions

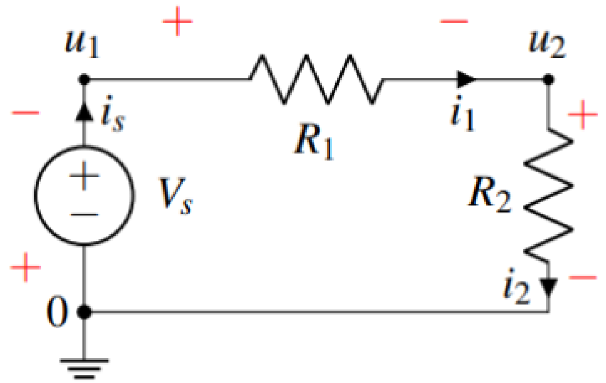
Step 4: Add +/− labels on each element to indicate positive/negative voltage, following the **passive sign convention** (defined below). These labels will indicate the direction with which voltage will be measured across that element.



Definition 3.4 (Passive Sign Convention): The **passive sign convention** dictates that positive current should *enter* the positive voltage terminal and *exit* the negative voltage terminal of an element. Below is an example for a resistor:



Write equations based on KCL and Ohm's law



Step 5: Identifying Unknowns and Reducing Them

At this stage in the circuit analysis algorithm, we find that there are several **unknowns** labeled on our circuit. These are the branch currents i_s, i_1, and i_2, and the node voltages u_1 and u_2.

We can reduce the number of unknown variables by making some plain substitutions.

- Simplify node voltages: We can apply the known voltage of the voltage source to reduce the number of unknown node voltages. For this example,

$$u_1 - 0 = V_s$$

thus the node voltage $u_1 = V_s$ is known.

Simplify element currents in the same branch: If a node connects only two elements, then applying KCL at this node reveals the currents through both elements are equivalent. In this example,

$$i_1 = i_s \quad \text{and} \quad i_2 = i_s$$

and so the circuit currents can be effectively expressed with a single unknown current i_s. Utilizing the current of a current source can also simplify the unknown element currents.

Write equations based on KCL and Ohm's law

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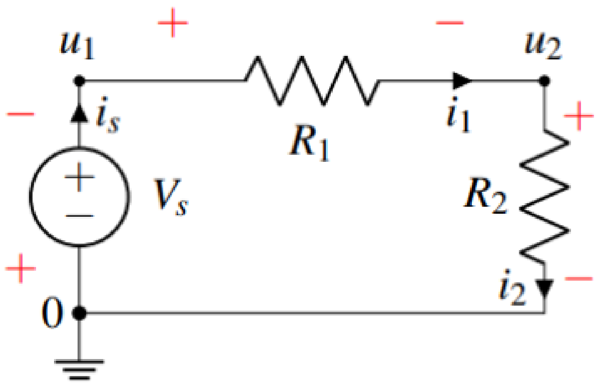
Step 6a: Set up a system of linear equations using KCL and I-V relationships. Write a KCL equation at each node with unknown voltage.
Begin by writing KCL equations for every node in the circuit.



$$i_s - i_1 = 0$$

$$i_1 - i_2 = 0$$

$$i_2 - i_s = 0$$

Write equations based on KCL and Ohm's law

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Step 6b: Use the I-V relationships of each element and express the voltage across each circuit element as a difference of node voltages. We know that the difference in potentials across the voltage source must be its voltage, V_s. We also know that the voltage across the resistor is equal to the current times the resistance, from Ohm's Law (i.e., $V = I \cdot R$). Note that the polarity of Ohm's Law depends on correctly applying the **passive sign convention.** Thus, we have the following equations:

$$V_{R1} = i_1 R_1$$

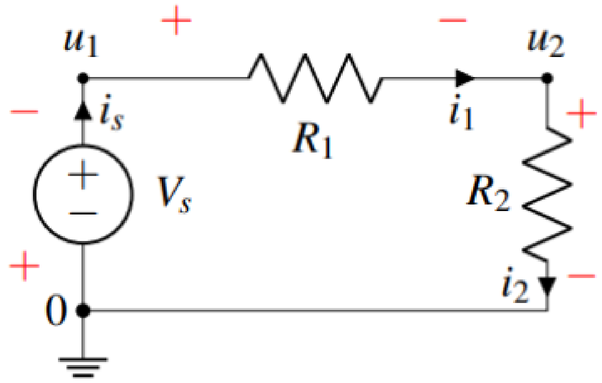
$$V_{R2} = i_2 R_2$$

Next, express the voltage across each circuit element as a difference of node voltages.

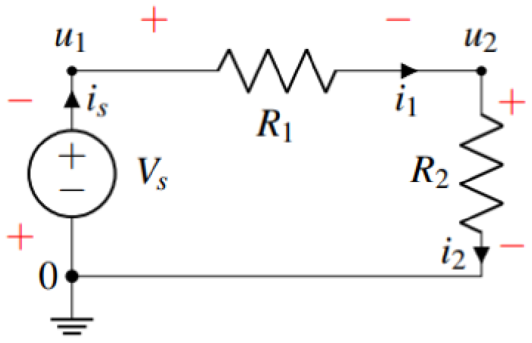
$$V_{R1} = u_1 - u_2$$

$$V_{R2} = u_2 - 0$$

In this way, we directly relate the element I-V characteristics and the node voltages.



Solve for unknowns



Step 7: Simplify your equations and solve. Be sure to incorporate the reduction in unknowns from Step 5 (i.e., $u_1 = V_s$, $i_1 = i_s$, $i_2 = i_s$).

The final equations in terms of the two unknowns u_2 and i_s are:

$$V_s - u_2 = i_s R_1$$

$$u_2 = i_s R_2$$

To solve, one can use substitution to find expressions for the unknowns u_2 and i_s:

$$u_2 = \frac{R_2}{R_1 + R_2} V_s$$

$$i_s = \frac{1}{R_1 + R_2} V_s$$

Touchpad – Network 2 in the demo board

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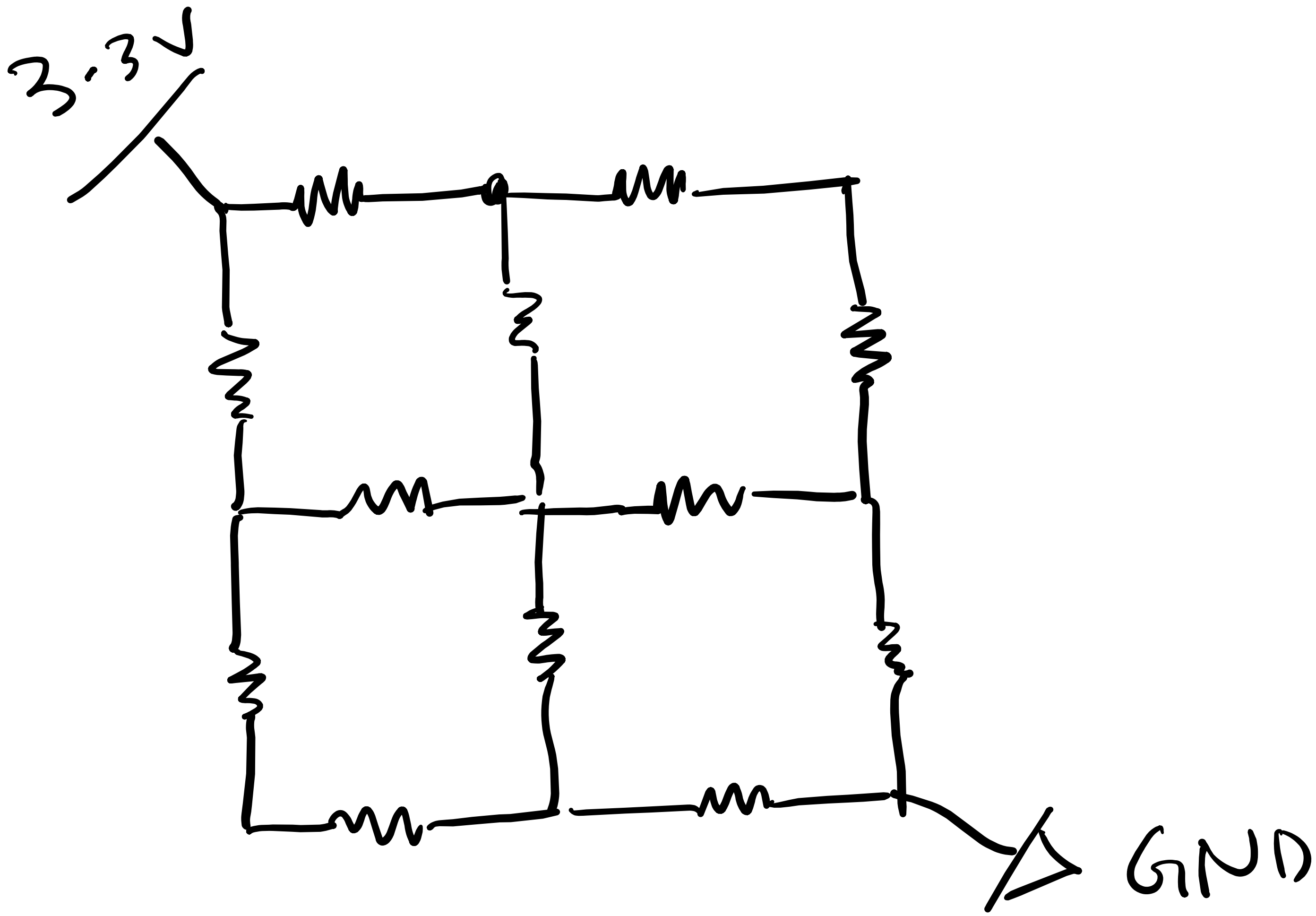

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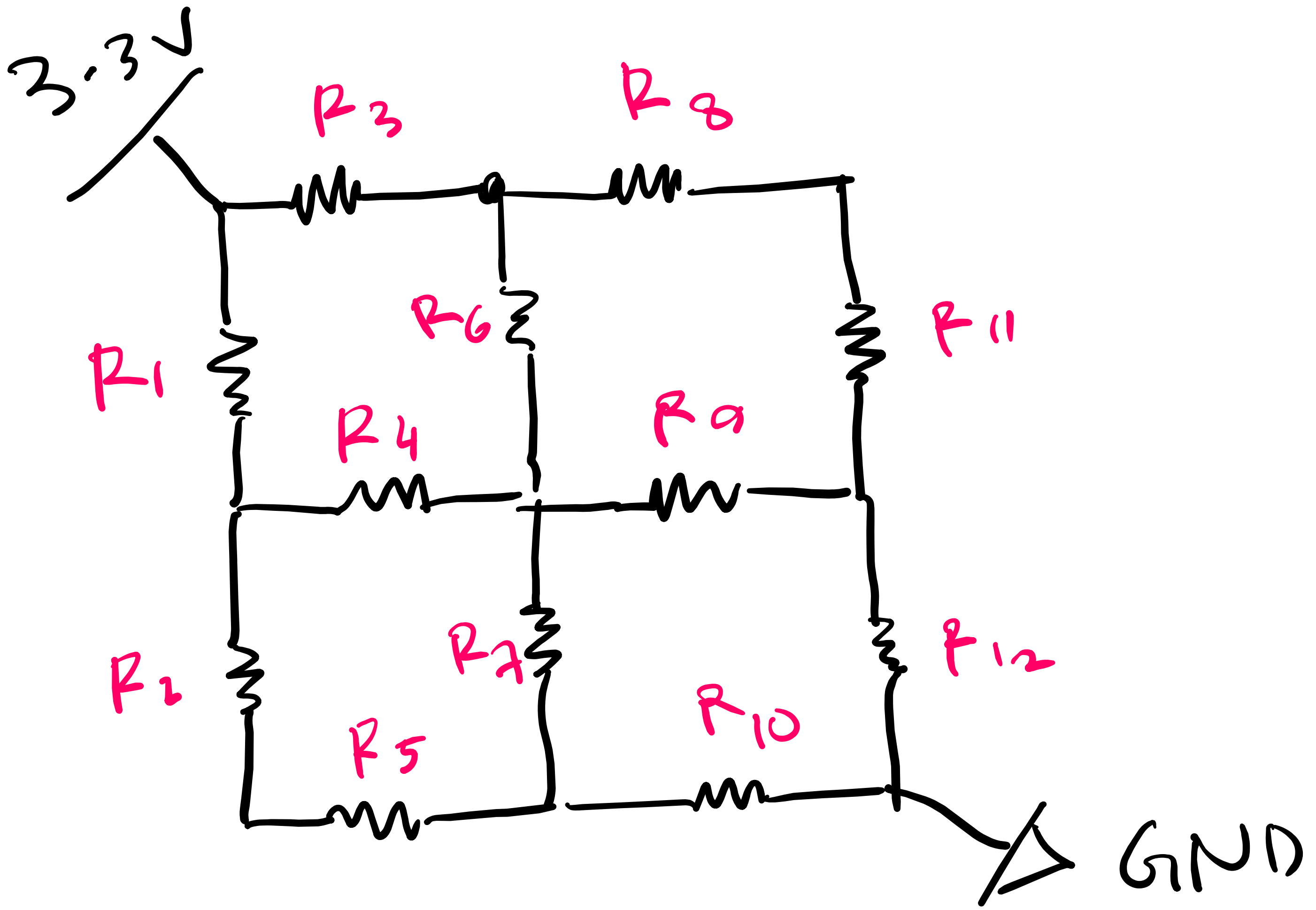

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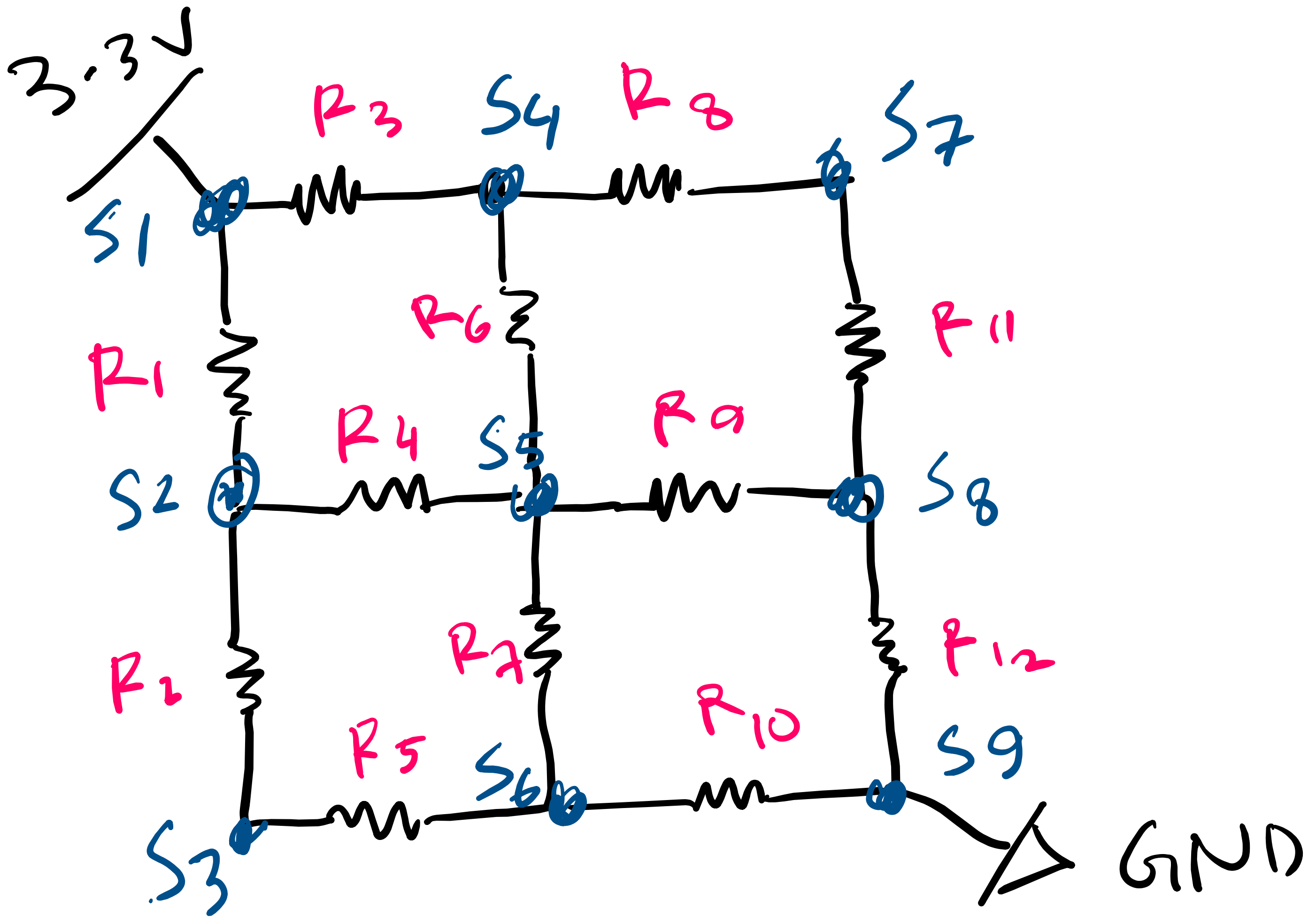

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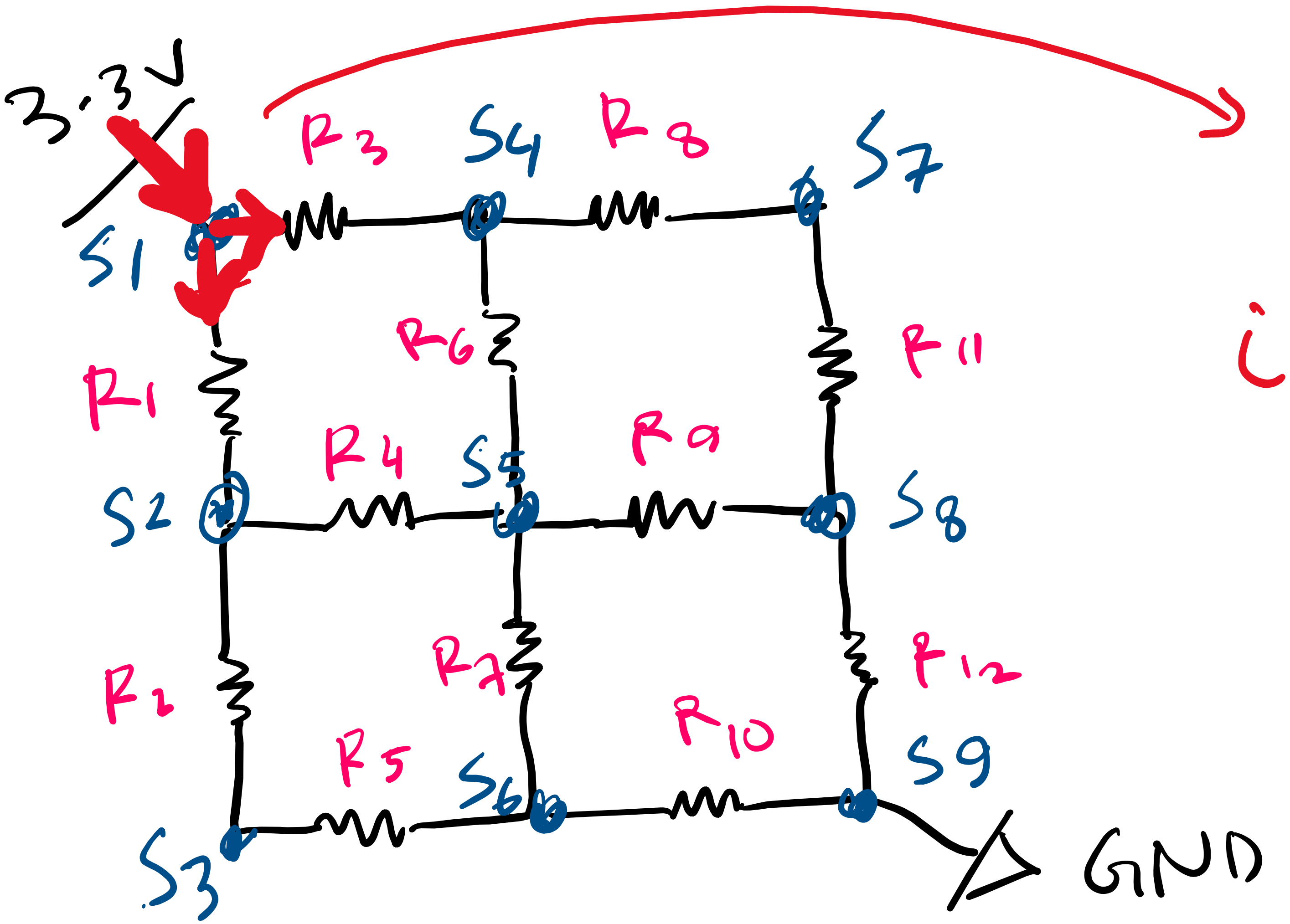


$$i_{tot} = i_{R_1} + i_{R_3}$$

$$i_1 = i_2 + i_4$$

$$i_3 = i_6 + i_8$$

$$\vdots$$

$$i_{10} + i_{12} = i_{tot}$$

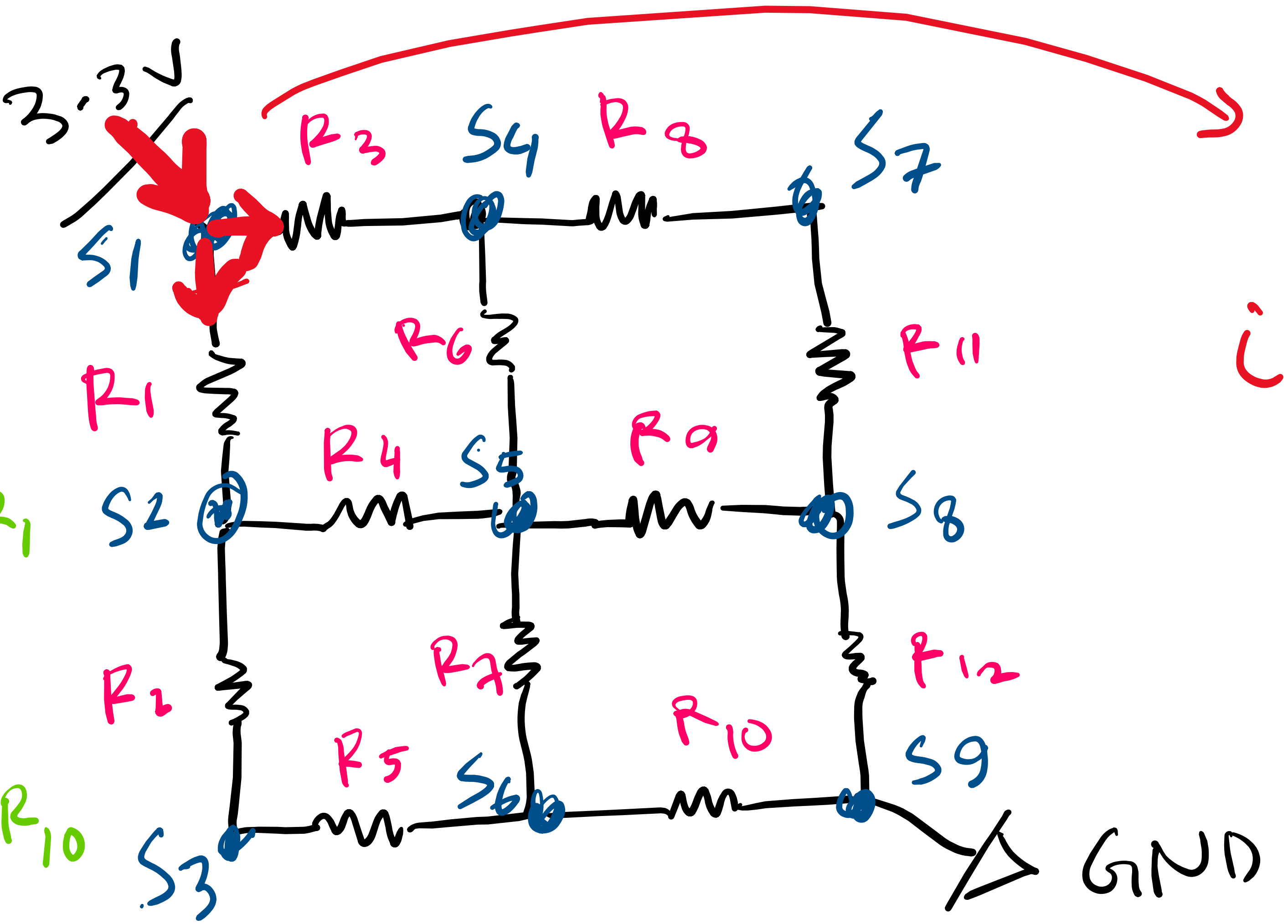
Touchpad – Network 2 in the demo board

$V = IR$

$S_1 - S_2 = i_1 R_1$

$\vdots$

$S_6 - S_9 = i_{10} R_{10}$



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$\vdots$

$i_{10} + i_{12} = i_{tot}$

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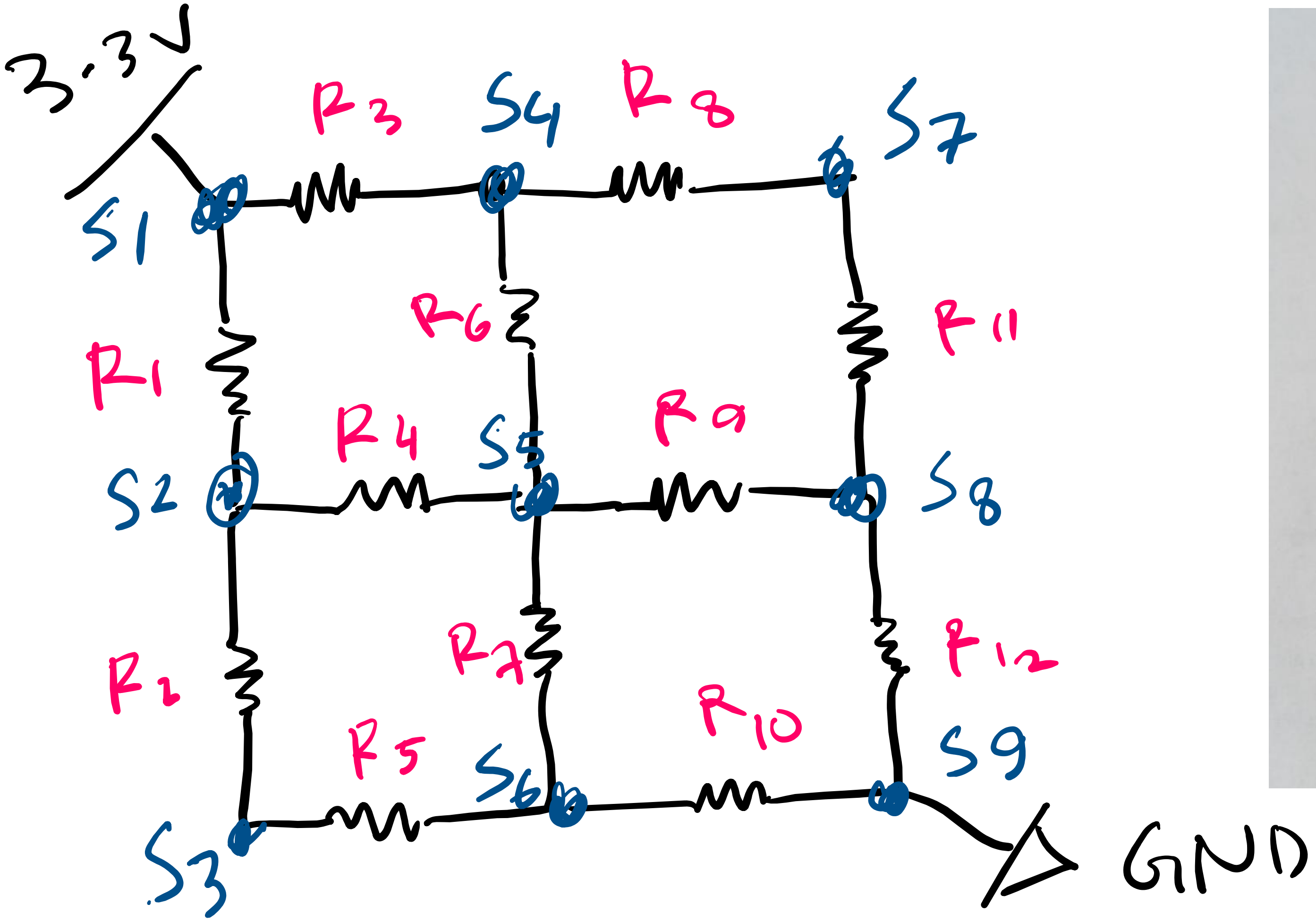

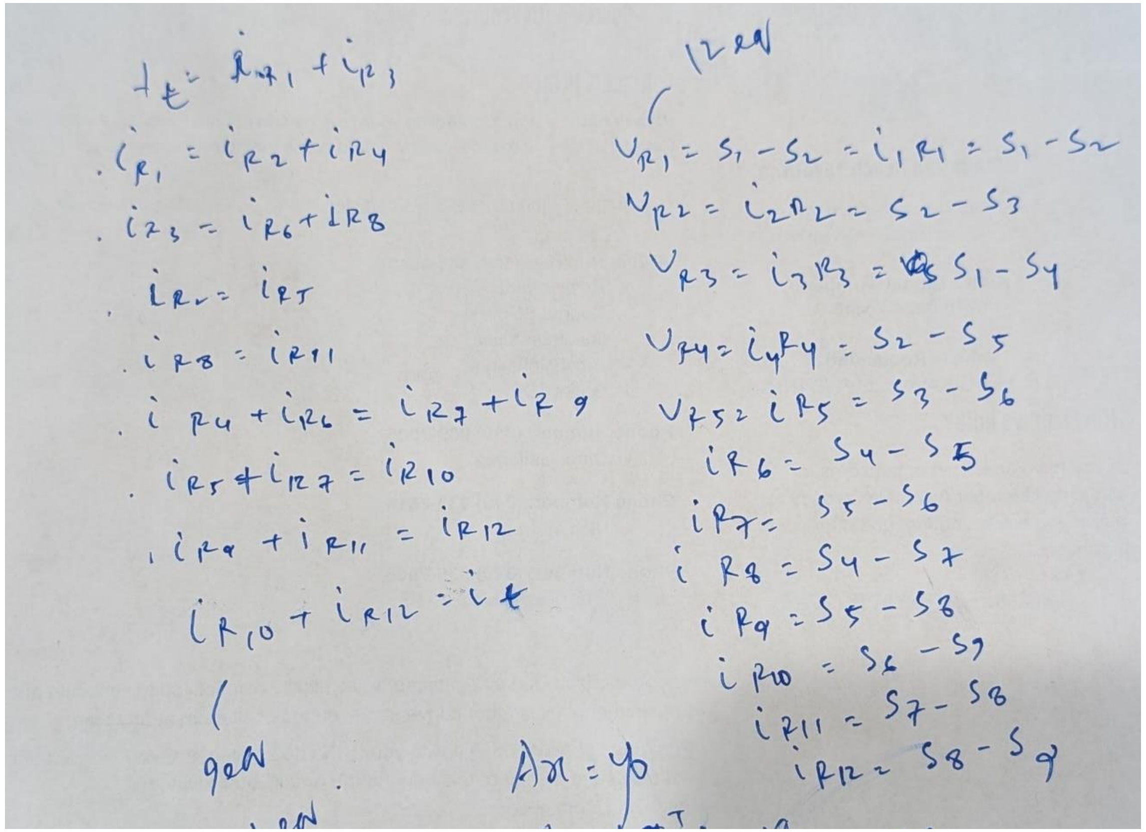

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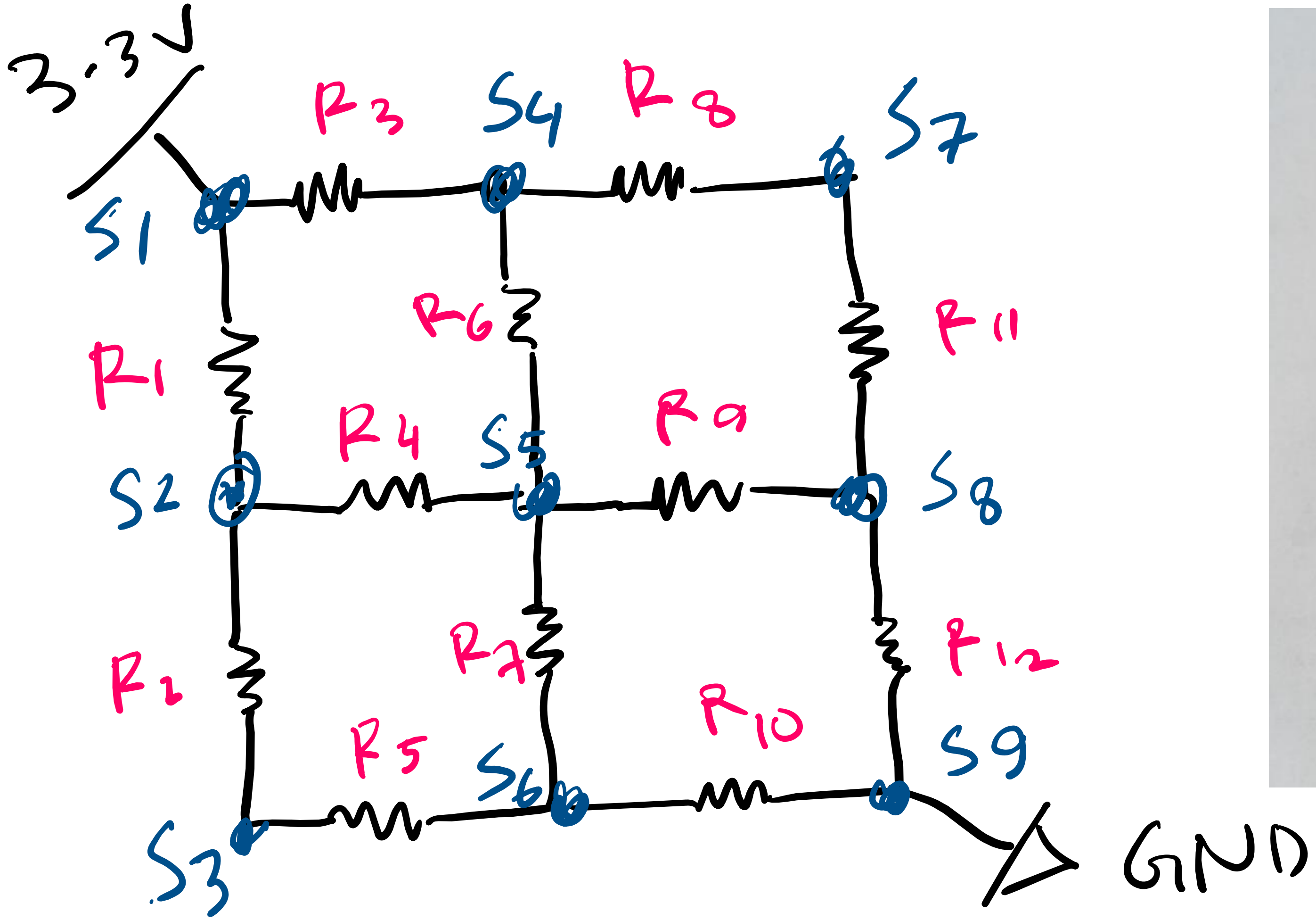

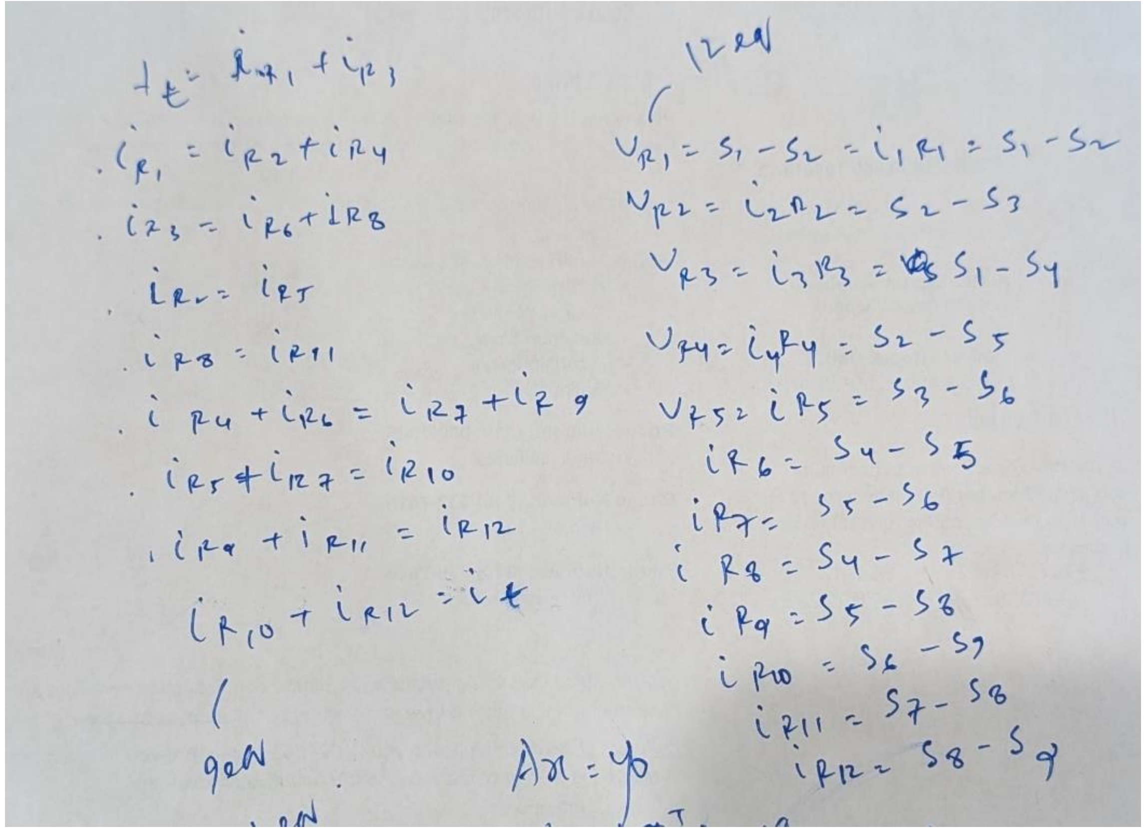

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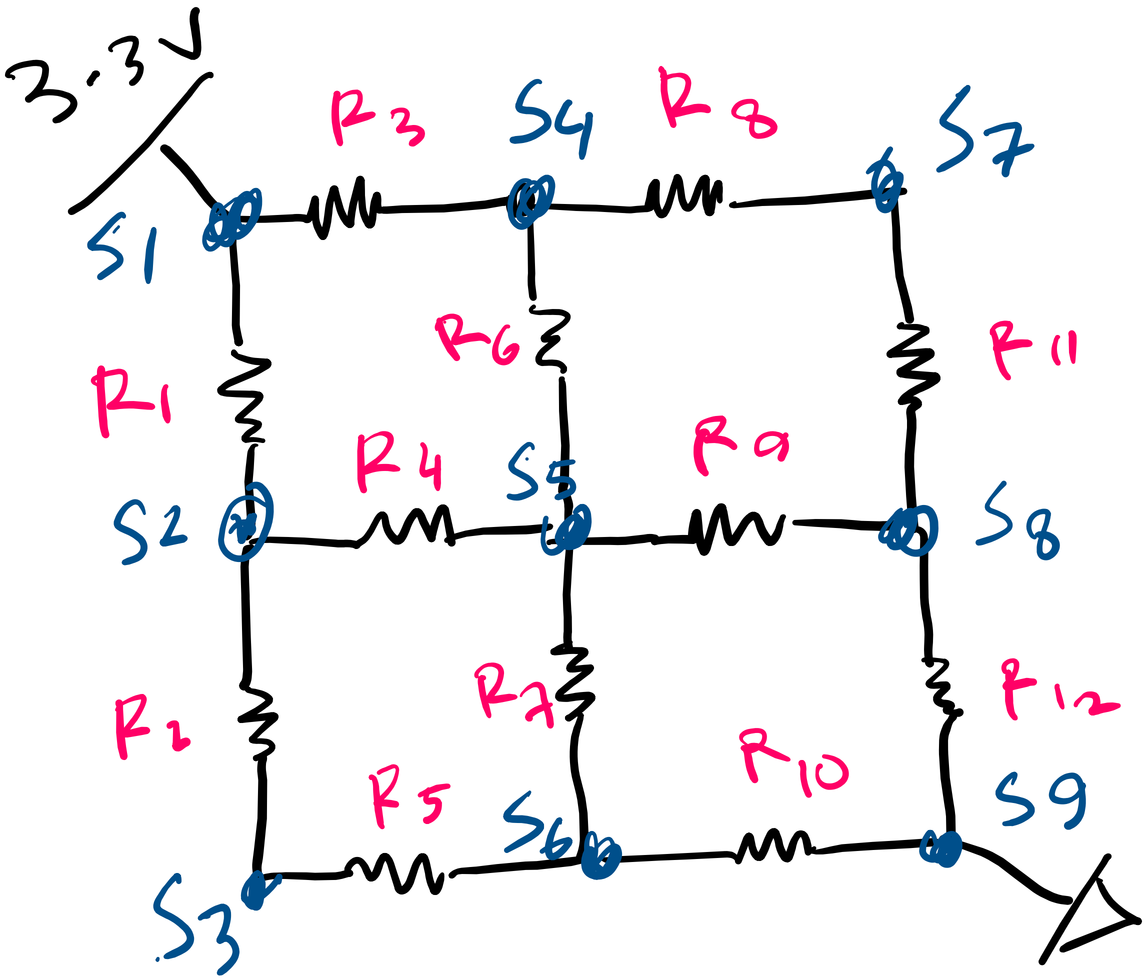
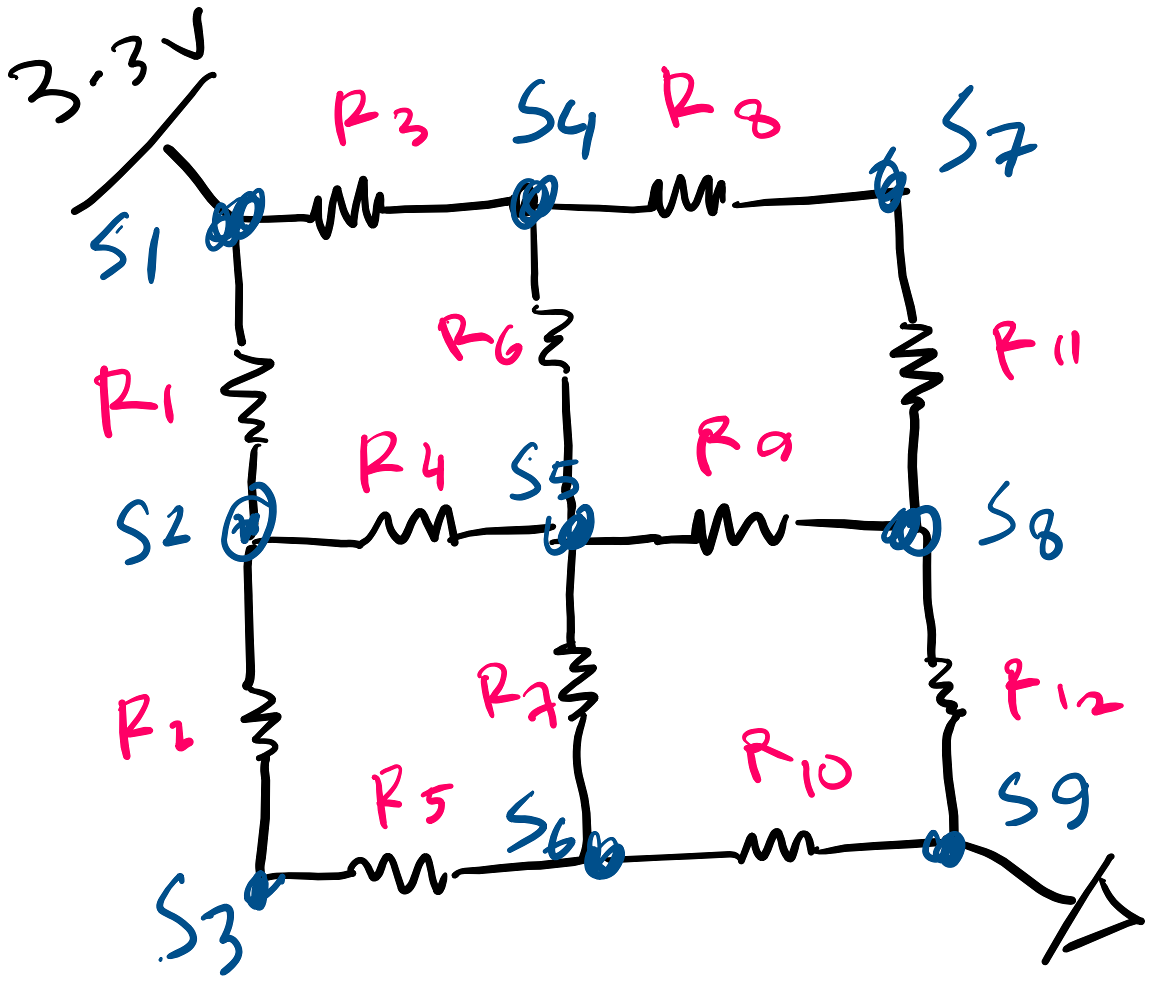


1. $it = i1 + i3$
2. $i1 = i2 + i4$
3. $i3 = i6 + i8$
4. $i2 = i5$
5. $i8 = i11$
6. $i4 + i6 = i7 + i9$
7. $i5 + i7 = i10$
8. $i9 + i11 = i12$
9. $i10 + i12 = it$
10. $i1 \cdot r1 = s1 - s2$
11. $i2 \cdot r2 = s2 - s3$
12. $i3 \cdot r3 = s1 - s4$
13. $i4 \cdot r4 = s2 - s5$
14. $i5 \cdot r5 = s3 - s6$
15. $i6 \cdot r6 = s4 - s5$
16. $i7 \cdot r7 = s5 - s6$
17. $i8 \cdot r8 = s4 - s7$
18. $i9 \cdot r9 = s5 - s8$
19. $i10 \cdot r10 = s6 - s9$
20. $i11 \cdot r11 = s7 - s8$
21. $i12 \cdot r12 = s8 - s9$

With the conditions:

- $r1 = r2 = r3 = r4 = r5 = r6 = r7 = r8 = r9 = r10 = r11 = r12 = 100$
- $s1 = 3.3$
- $s9 = 0$

Touchpad – Network 2 in the demo board



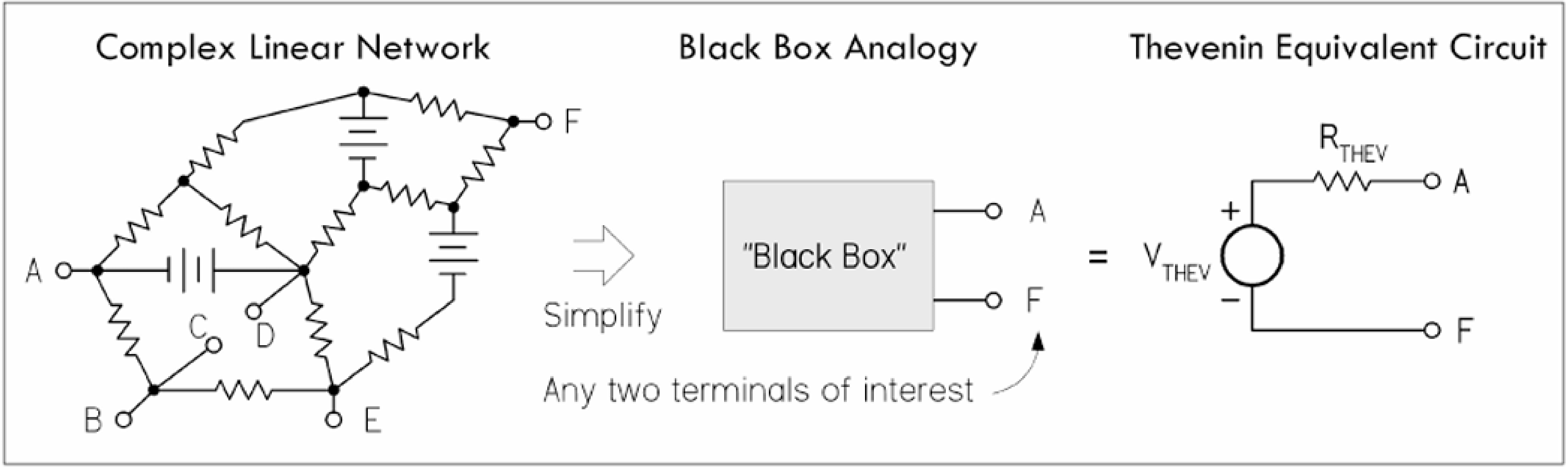
The solution to the system of equations is as follows:

- $i1 = 0.011$
- $i2 = 0.0055$
- $i3 = 0.011$
- $i4 = 0.0055$
- $i5 = 0.0055$
- $i6 = 0.0055$
- $i7 = 0.0055$
- $i8 = 0.0055$
- $i9 = 0.0055$
- $i10 = 0.011$
- $i11 = 0.0055$
- $i12 = 0.011$
- $it = 0.022$

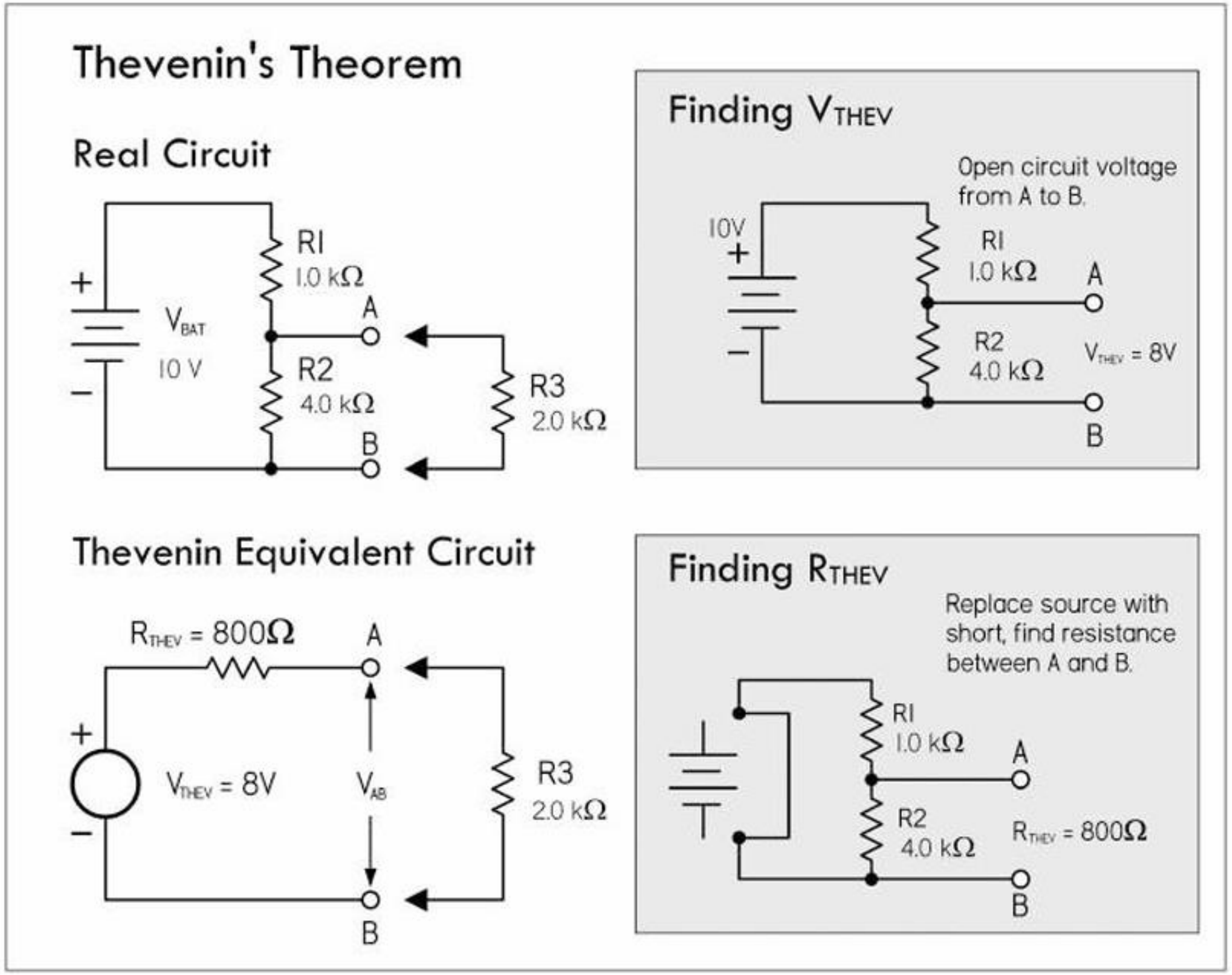
The intermediate variables are:

- $s2 = 2.2$
- $s3 = 1.65$
- $s4 = 2.2$
- $s5 = 1.65$
- $s6 = 1.1$
- $s7 = 1.65$
- $s8 = 1.1$

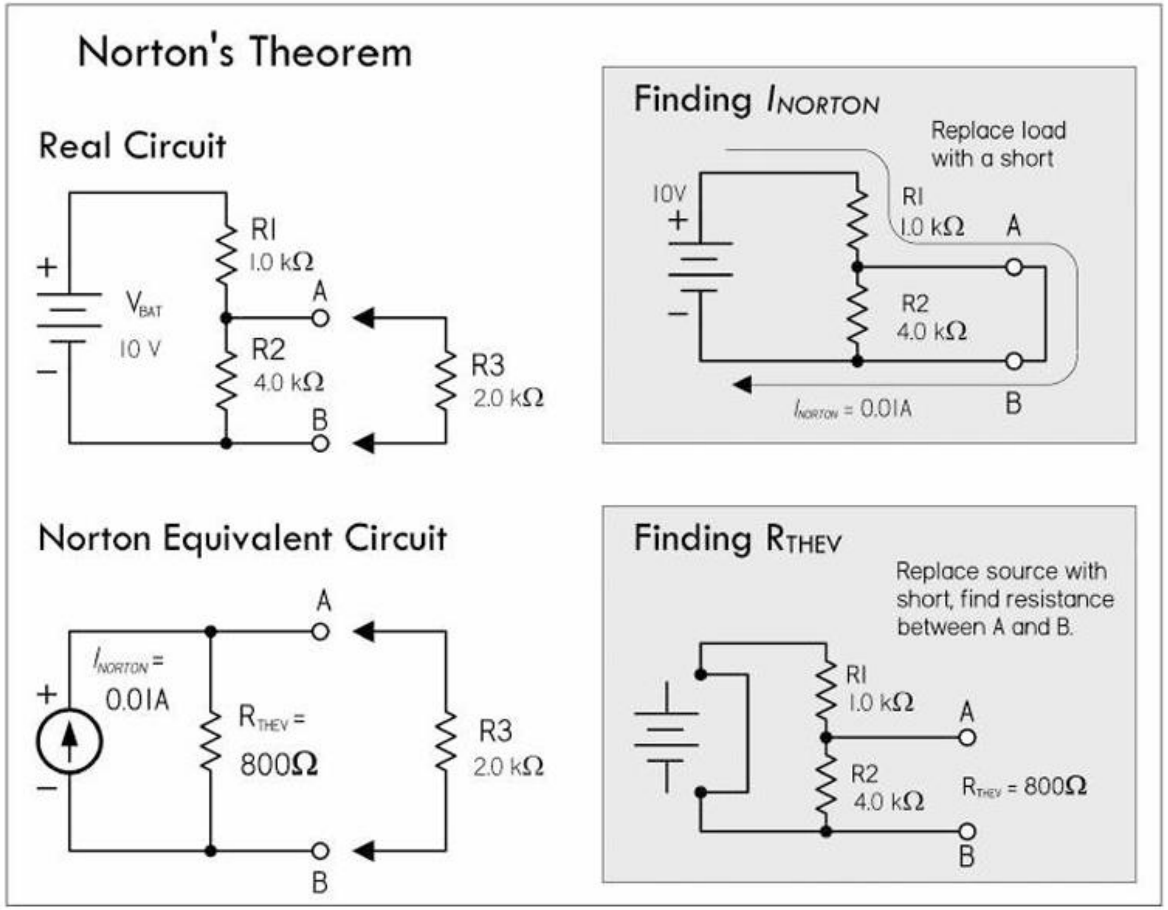
Equivalent circuits



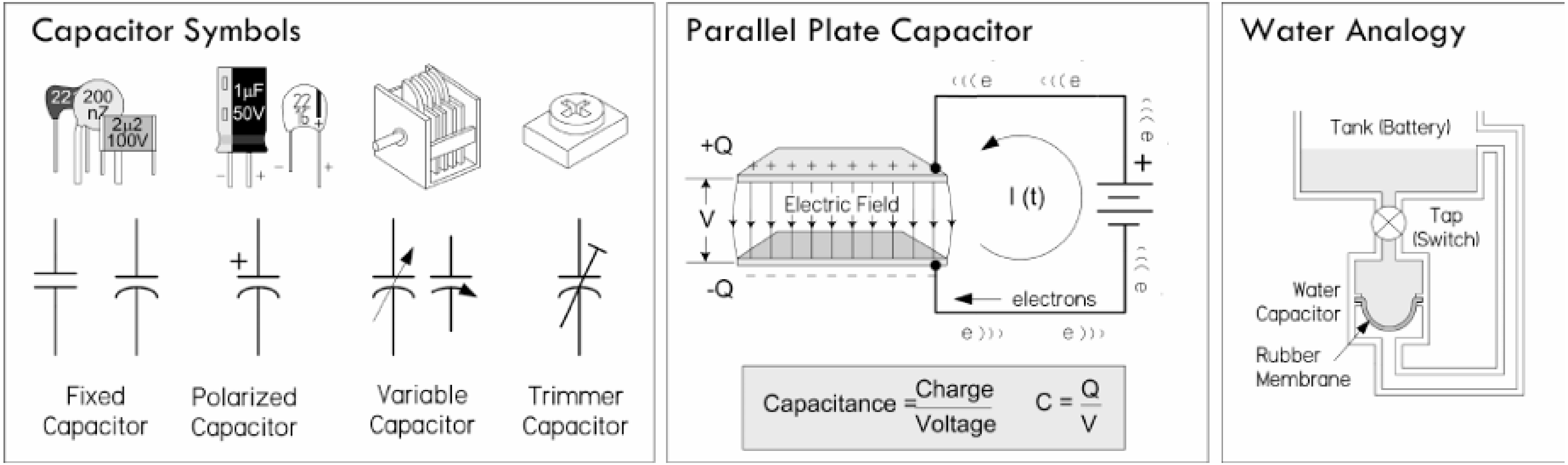
Thevenin's equivalent circuit



Norton’s equivalent circuit



Capacitors



Capacitor Symbols
Fixed Capacitor
Polarized Capacitor
Variable Capacitor
Trimmer Capacitor
Parallel Plate Capacitor
+Q
-Q
V
Electric Field
I (t)
electrons
Capacitance = Charge / Voltage
C = Q / V
Water Analogy
Tank (Battery)
Tap (Switch)
Water Capacitor
Rubber Membrane

Capacitors

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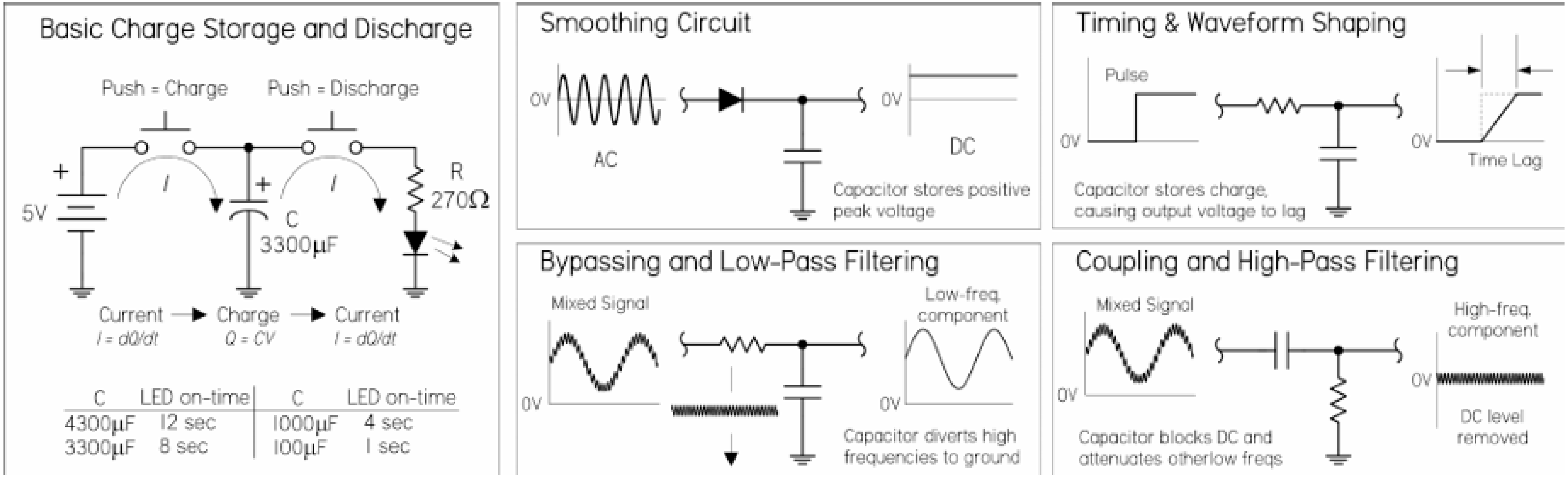
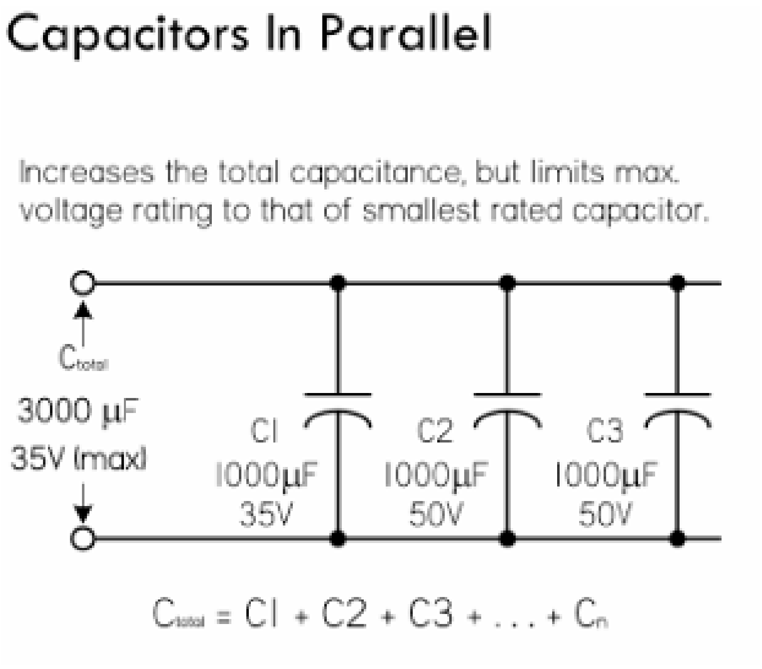
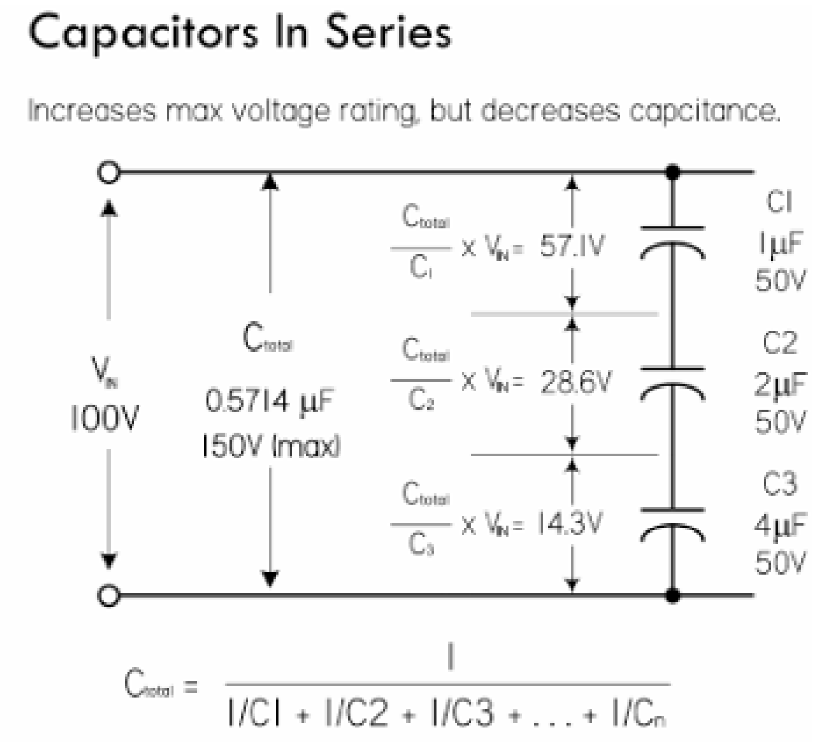
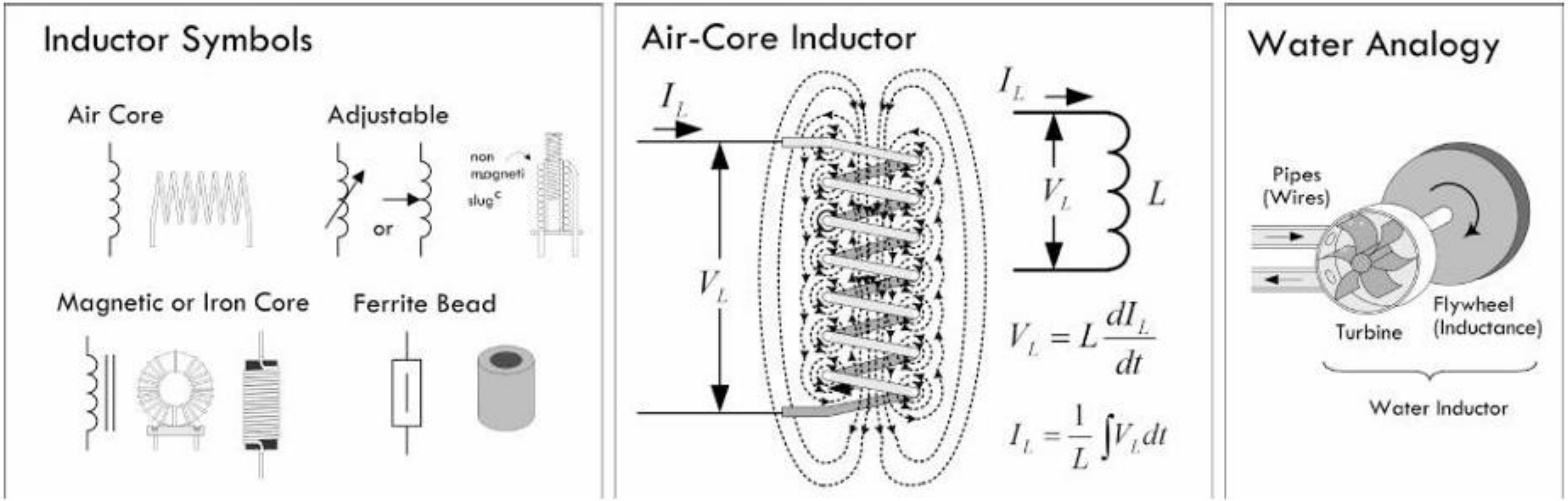


Capacitors

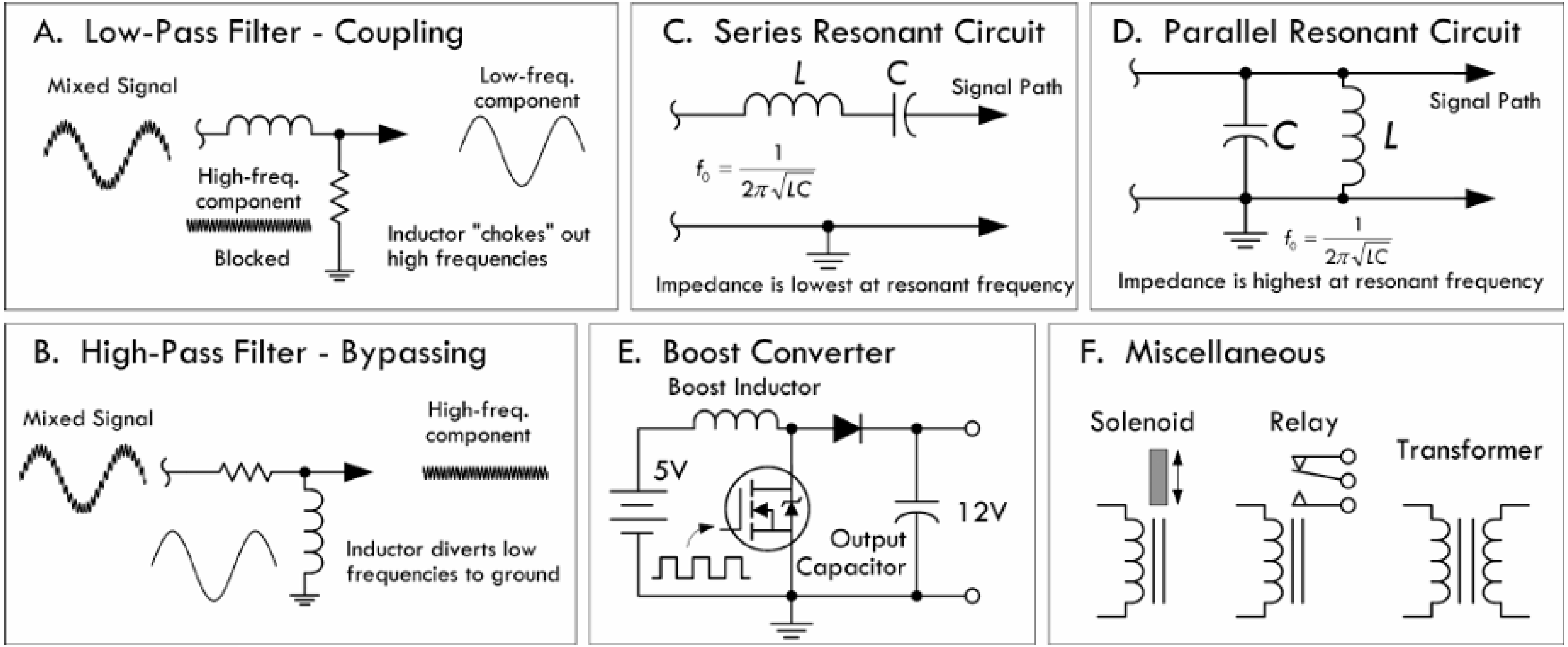

Capacitors In Parallel
Increases the total capacitance, but limits max. voltage rating to that of smallest rated capacitor.
Ctotal
3000 µF
35V (max)
C1
1000µF
35V
C2
1000µF
50V
C3
1000µF
50V
Ctotal = C1 + C2 + C3 + . . . + Cn


Capacitors In Series
Increases max voltage rating, but decreases capcitance.
VIN
100V
Ctotal
0.5714 µF
150V (max)
Ctotal / C1 x VIN = 57.1V
Ctotal / C2 x VIN = 28.6V
Ctotal / C3 x VIN = 14.3V
C1
1µF
50V
C2
2µF
50V
C3
4µF
50V
Ctotal = 1 / (1/C1 + 1/C2 + 1/C3 + . . . + 1/Cn)

Inductors


Inductor Symbols
Air Core
Adjustable
or
Magnetic or Iron Core
Ferrite Bead
Air-Core Inductor
I_L
V_L
L
$V_L = L \frac{dI_L}{dt}$
$I_L = \frac{1}{L} \int V_L dt$
Water Analogy
Pipes (Wires)
Turbine
Flywheel (Inductance)
Water Inductor

Inductors



A. Low-Pass Filter - Coupling
Mixed Signal
Low-freq. component
High-freq. component
Blocked
Inductor "chokes" out high frequencies
C. Series Resonant Circuit
L
C
Signal Path
$f_0 = \frac{1}{2\pi\sqrt{LC}}$
Impedance is lowest at resonant frequency
D. Parallel Resonant Circuit
Signal Path
C
L
$f_0 = \frac{1}{2\pi\sqrt{LC}}$
Impedance is highest at resonant frequency
B. High-Pass Filter - Bypassing
Mixed Signal
High-freq. component
Inductor diverts low frequencies to ground
E. Boost Converter
Boost Inductor
5V
12V
Output Capacitor
F. Miscellaneous
Solenoid
Relay
Transformer

Inductors

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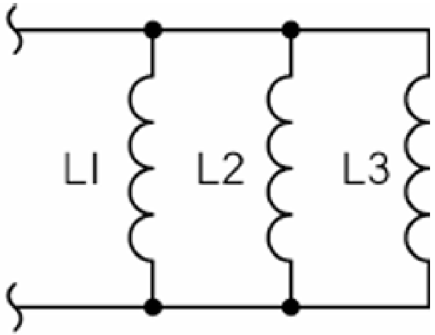
Inductors in Series



Inductors in parallel