Practice Midterm B SOLUTIONS

Modules 1 and 2 · Lectures 5 to 7, 9 and 10

Instructions

- Time: 120 minutes. Total: 100 points.
- Part I has 8 short problems (50 points). Plan about 45 minutes for Part I.
- Part II has 4 long problems (50 points). Plan about 65 minutes for Part II. Keep about 10 minutes to check your answers.
- Do not use a calculator. The numbers in each problem need only simple mental arithmetic.
- Show your method for each answer. Give the unit with each numerical answer.
- Where a problem tells you to leave an expression in a given form, do not reduce it further. You can leave a result such as 4/3 mA as a fraction.
- All wires and sources are ideal, unless a problem gives other data.
- Power: an element absorbs $P = V \cdot I$ when its current enters its "+" terminal (passive sign convention). When the current leaves the "+" terminal, the element supplies $P = V \cdot I$.
- Notation: V_T is the MOSFET threshold voltage. kT/q is the thermal voltage in the diode equation. V_{TH} is a Thevenin voltage. I_s is the reverse saturation current of a diode.
- With resistances in kΩ and voltages in V, currents come out in mA and powers in mW.

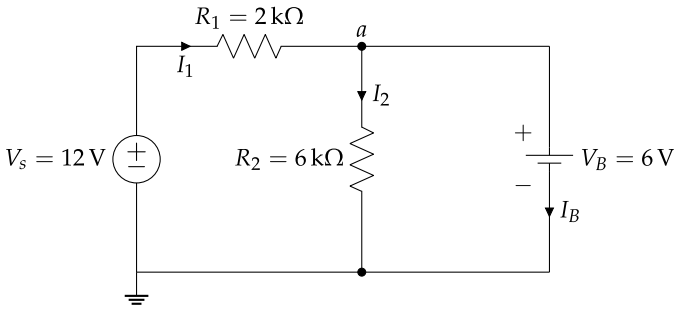
Constants

- $q = 1.602 \times 10^{-19}$ C. Use $q \approx 1.6 \times 10^{-19}$ C in hand calculations.
- $n_i = 10^{10}$ cm^{-3} for Si at 300 K, and $n \cdot p = n_i^2$.
- $kT/q = 0.026$ V at 300 K.
- $\ln 10 \approx 2.3$, so $(kT/q) \cdot \ln 10 \approx 0.06$ V.
- Photon energy: $E\ (\text{eV}) = 1240 / \lambda\ (\text{nm})$.
- Band gaps (L9-16): Ge 0.67 eV, Si 1.12 eV, GaAs 1.42 eV, GaP 2.25 eV, diamond 6.0 eV.
- Lengths: 1 cm = 10 mm = 10^4 μm.
- Prefixes: f = 10^{-15}, n = 10^{-9}, μ = 10^{-6}, m = 10^{-3}, k = 10^3, M = 10^6.

Part I · Short problems (50 points)

Problem 1 CIRCUITS EASY 6 POINTS

A 12 V source charges a 6 V battery V_B through R_1. R_2 is in parallel with the battery.



(a) Find the voltage V_a at node a and the current I_1 in R_1. (2 points)

(b) Find the current I_2 in R_2 and the battery current I_B. State whether I_B enters or leaves the "+" terminal of the battery, and whether the battery charges or discharges. (2 points)

(c) Find the power of each source (V_s and the battery). State whether each source absorbs or supplies power. (2 points)

SOLUTION

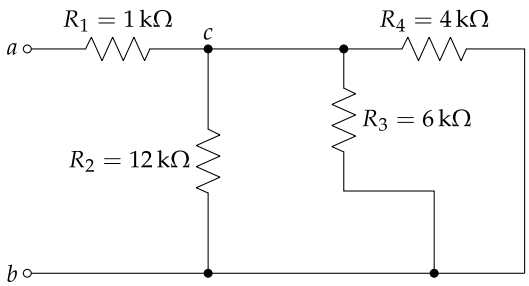
(a) The battery connects node a to ground, so $\mathbf{V_a = V_B = 6\ V}$: a voltage source fixes the voltage across it for any current (L5-7). KVL: R_1 has 12 V − 6 V = 6 V across it, so $\mathbf{I_1 = 6\ V\ /\ 2\ k\Omega = 3\ mA}$, from the source toward node a.

(b) $\mathbf{I_2 = 6\ V\ /\ 6\ k\Omega = 1\ mA}$, down through R_2. KCL at node a (L5-9): $I_1 = I_2 + I_B$, so $\mathbf{I_B = 3\ mA - 1\ mA = 2\ mA}$. **$I_B$ enters the "+" terminal of the battery, so the battery charges.**

(c) The current leaves the "+" terminal of V_s, so $\mathbf{V_s}$ **supplies 12 V × 3 mA = 36 mW**. The current enters the "+" terminal of the battery, so **the battery absorbs 6 V × 2 mA = 12 mW** (passive sign convention, L6-17). Check: R_1 absorbs 6 V × 3 mA = 18 mW and R_2 absorbs 6 V × 1 mA = 6 mW, so 18 mW + 6 mW + 12 mW = 36 mW, which equals the power supplied.

Problem 2 CIRCUITS EASY 6 POINTS

The resistor network has two terminals, a and b.



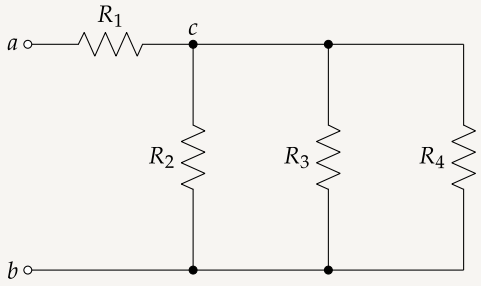
(a) How many distinct nodes does the network have? (Wires that connect form one node.) Which resistors connect the same pair of nodes? (2 points)

(b) Find the equivalent resistance R_{ab}. (3 points)

(c) A 9 V source is connected across a–b, with "+" at a. Find the source current. (1 point)

SOLUTION

The redrawn network shows that R_2, R_3 and R_4 share the nodes c and b:



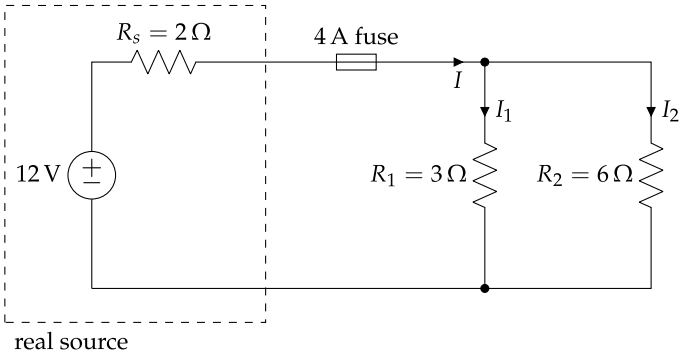
(a) Follow the wires from terminal b (L5-17 to L5-19). The bottom of R_2, the short wire below R_3 and the wire at the right end of R_4 all reach the bottom wire, which is terminal b. The upper ends of R_2, R_3 and R_4 all sit on the wire to the right of R_1, which is node c. So **the network has 3 distinct nodes: a, c and b. R_2, R_3 and R_4 all connect c to b, so they are in parallel** (L6-8).

(b) $1/R_p = 1/(12\ \text{k}\Omega) + 1/(6\ \text{k}\Omega) + 1/(4\ \text{k}\Omega) = (1 + 2 + 3)/(12\ \text{k}\Omega) = 6/(12\ \text{k}\Omega)$, so $R_p = 2\ \text{k}\Omega$ (L5-13). R_1 is in series with this group: $\mathbf{R_{ab} = 1\ k\Omega + 2\ k\Omega = 3\ k\Omega}$.

(c) **Source current = 9 V / 3 kΩ = 3 mA.** (Not asked: the parallel group has 3 mA × 2 kΩ = 6 V across it, so R_2, R_3 and R_4 carry 0.5 mA, 1 mA and 1.5 mA.)

Problem 3 CIRCUITS EASY 6 POINTS

A real 12 V source has an internal resistance $R_s = 2\ \Omega$. It feeds two loads, $R_1 = 3\ \Omega$ and $R_2 = 6\ \Omega$, through a 4 A fuse.



(a) Find the source current I and the voltage across the loads. Does the fuse blow? (2 points)

(b) Use the current divider to find I_1 and I_2. (2 points)

(c) R_2 fails and becomes a short circuit (0 Ω). Find the new current I, and state whether the fuse blows. (2 points)

SOLUTION

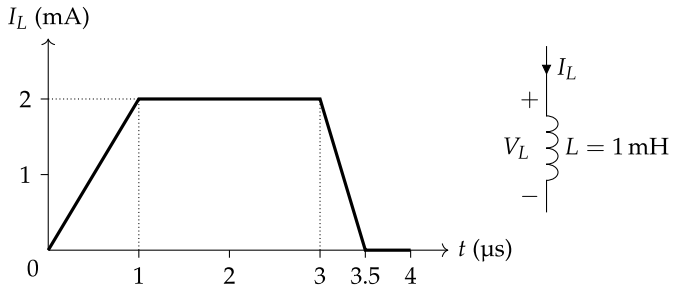
(a) The real source is an ideal 12 V source in series with R_s (L5-3). $R_1 \parallel R_2 = (3 \times 6)/(3 + 6)\ \Omega = 2\ \Omega$. The total resistance is $R_s + 2\ \Omega = 4\ \Omega$, so $\mathbf{I = 12\ V / 4\ \Omega = 3\ A}$. **The load voltage is 3 A × 2 Ω = 6 V. 3 A is less than 4 A, so the fuse does not blow.**

(b) Current divider (L5-14): $\mathbf{I_1 = I{\cdot}R_2/(R_1 + R_2) = 3\ A \times 6/9 = 2\ A}$ and $\mathbf{I_2 = I{\cdot}R_1/(R_1 + R_2) = 3\ A \times 3/9 = 1\ A}$. Check with Ohm's law: 6 V / 3 Ω = 2 A and 6 V / 6 Ω = 1 A.

(c) The short puts 0 Ω across both loads, so only R_s limits the current: **I = 12 V / 2 Ω = 6 A. 6 A is more than 4 A, so the fuse blows** and the current stops (L5-25). The fuse sits in series with the source, so it carries the total current and opens the circuit for a fault in any load.

Problem 4 CIRCUITS EASY 7 POINTS

An inductor L = 1 mH carries the current I_L in the graph. I_L enters the "+" terminal of V_L.



(a) Find V_L in each interval: 0 to 1 μs, 1 μs to 3 μs, and 3 μs to 3.5 μs. (3 points)

(b) Two capacitors, $C_1 = 1$ μF and $C_2 = 3$ μF, start uncharged. They are connected in series across a 4 V source. Find C_{eq} and the voltage across each capacitor. (2 points)

(c) Each of these capacitors is rated 6 V. Find the largest source voltage that keeps both capacitors within their ratings. (2 points)

SOLUTION

(a) $V_L = L \cdot dI_L/dt$ (L6-36), and dI_L/dt is the slope of the graph. **0 to 1 μs:** the slope is 2 mA / 1 μs = 2000 A/s, so $\mathbf{V_L = 10^{-3}\ H \times 2000\ A/s = 2\ V}$. **1 μs to 3 μs:** the current is constant, so $\mathbf{V_L = 0\ V}$. **3 μs to 3.5 μs:** the slope is −2 mA / 0.5 μs = −4000 A/s, so $\mathbf{V_L = -4\ V}$. The negative value means that the "−" terminal is now at the higher potential: the inductor opposes the fall of its current.

(b) Series capacitors combine like parallel resistors (L6-35): $\mathbf{C_{eq} = (1 \times 3)/(1 + 3)\ \mu F = 0.75\ \mu F}$. The charge is $Q = C_{eq} \cdot V = 0.75\ \mu F \times 4\ V = 3\ \mu C$, and both capacitors hold this charge. $\mathbf{V_1 = 3\ \mu C / 1\ \mu F = 3\ V}$ and $\mathbf{V_2 = 3\ \mu C / 3\ \mu F = 1\ V}$. KVL check: 3 V + 1 V = 4 V. The slide formula $V_k = (C_{total}/C_k) \cdot V_{in}$ gives the same $V_1 = (0.75/1) \times 4\ V = 3\ V$.

(c) The smaller capacitance (C_1) takes the larger share of the voltage: $V_1 = (C_{eq}/C_1) \cdot V = (3/4) \cdot V$. The condition $V_1 \le 6$ V gives $\mathbf{V \le 8\ V}$. At V = 8 V, $V_2 = 2$ V, which is also within the rating. So the limit is 8 V, not the sum of the two ratings (12 V).

Problem 5 DEVICES EASY 6 POINTS

A Si sample is doped with boron, $N_A = 10^{17}\ cm^{-3}$. Use $\mu_p = 250\ cm^2/V \cdot s$.

(a) State whether boron is a donor or an acceptor in Si. State whether the sample is n-type or p-type. Name the majority carrier. (1 point)

(b) Find p and n. (2 points)

(c) Find the conductivity σ and the resistivity ρ. Explain why you can ignore the electrons. (3 points)

SOLUTION

(a) Boron is in group III. It has 3 valence electrons, so one of its 4 bonds lacks an electron, and this missing electron is a hole (L9-13). So boron is an **acceptor**, the sample is **p-type**, and the **majority carriers are holes**.

(b) Each acceptor gives one hole: $\mathbf{p \approx N_A = 10^{17}\ cm^{-3}}$. From $n \cdot p = n_i^2$: $\mathbf{n = 10^{20}/10^{17} = 10^3\ cm^{-3}}$ (L9-18 to L9-20).

(c) $\mathbf{\sigma = q \cdot \mu_p \cdot p = 1.6 \times 10^{-19} \times 250 \times 10^{17}\ S/cm = 4\ S/cm}$ (because $1.6 \times 250 = 1600/4 = 400$). $\mathbf{\rho = 1/\sigma = 0.25\ \Omega \cdot cm}$. **n is 10^{14} times smaller than p**, so the electron term $q \cdot \mu_n \cdot n$ is negligible, although electrons are more mobile than holes.

Problem 6 DEVICES EASY 6 POINTS

A Si diode has $I_s = 2 \times 10^{-12}$ A and an ideality factor of 1. V is the diode voltage (anode minus cathode), and I is the diode current from anode to cathode.

(a) Find I at V = 0.52 V. First write I in the form $I_s \cdot e^{(\cdots)}$. Then use $e^{20} \approx 4.85 \times 10^8$. (2 points)

(b) By what factor does I increase when V increases by 60 mV? Find I at V = 0.58 V. (2 points)

(c) Find I at V = −3 V. State which terminal is more positive in this case, and explain why the current is so small. (2 points)

SOLUTION

(a) $V/(kT/q) = 0.52/0.026 = 20$, and e^{20} is much larger than 1, so the "−1" term is negligible (L9-28): $\mathbf{I \approx 2 \times 10^{-12} \times e^{20}\ A}$. With $e^{20} \approx 4.85 \times 10^8$: $\mathbf{I \approx 2 \times 10^{-12} \times 4.85 \times 10^8\ A = 9.7 \times 10^{-4}\ A \approx 0.97\ mA}$. The anode is positive, so the diode is forward biased: holes cross from the p side to the n side, and electrons cross from the n side to the p side (L9-23).

(b) I is proportional to $e^{V/(kT/q)}$. An increase of 60 mV is an increase of $(kT/q) \cdot \ln 10$, so it multiplies I by $e^{\ln 10} = \mathbf{10}$. V = 0.58 V is 60 mV above 0.52 V, so $\mathbf{I \approx 10 \times 0.97\ mA = 9.7\ mA}$.

(c) $e^{-3/0.026}$ is practically zero, so $\mathbf{I \approx -I_s = -2 \times 10^{-12}\ A}$. **The cathode (n side) is 3 V more positive than the anode, so the diode is reverse biased.** The reverse voltage pulls holes and electrons away from the junction, and the depletion region grows. Only fixed dopant ions stay in it, so only a tiny leakage current flows. This current does not change with the reverse voltage until breakdown (L9-24, L9-30).

Problem 7 DEVICES EASY 7 POINTS

A PMOS transistor has $k' = \mu_p C_{ox}(W/L) = 1\ mA/V^2$ and $V_T = -1$ V. Its source is at $V_{DD} = 5$ V. Use the NMOS current equations with all signs reversed: use V_{SG} for V_{GS}, V_{SD} for V_{DS} and $|V_T|$ for V_T. The current flows from source to drain.

(a) Find the range of gate voltages V_G that turn the PMOS on. (2 points)

(b) $V_G = 2$ V and $V_D = 4$ V. Find the region and the current. (3 points)

(c) $V_G = 2$ V and $V_D = 1$ V. Find the region and the current. (2 points)

SOLUTION

(a) The PMOS is on when $V_{GS} < V_T$, that is, $V_G - 5\ V < -1\ V$. So **the PMOS is on for $\mathbf{V_G < 4\ V}$** (L10-19, L10-20).

(b) $V_{SG} = 5\ V - 2\ V = 3\ V$ and $V_{SD} = 5\ V - 4\ V = 1\ V$, so $V_{SG} - |V_T| = 2$ V. $V_{SD} = 1$ V is less than 2 V, so the PMOS is in the **linear region** (L10-22). $\mathbf{I_D = 1\ mA/V^2 \times [2 \times 1 - ½ \times 1^2]\ V^2 = 1.5\ mA}$, from source to drain.

(c) $V_{SD} = 5\ V - 1\ V = 4$ V, which is at least 2 V, so the PMOS is in **saturation**. $\mathbf{I_D = ½ \times 1\ mA/V^2 \times (2\ V)^2 = 2\ mA}$. An NMOS with the same W/L would carry about twice this current, because electrons have about twice the mobility of holes (L10-32).

Problem 8 DEVICES EASY 6 POINTS

An NMOS transistor has $V_T = 1$ V. Its gate is at $V_G = 5$ V, its source is at $V_S = 0$ V, and its drain is at $V_D = 2$ V.

(**a**) Find the voltage across the oxide at the source end and at the drain end of the channel. (2 points)

(**b**) The channel charge per unit area is proportional to (oxide voltage $- V_T$). Find the ratio of the charge at the drain end to the charge at the source end. (2 points)

(**c**) At what V_D does the channel pinch off at the drain end? In one or two sentences, explain why I_D stays almost constant when V_D rises above this value. (2 points)

SOLUTION

(**a**) The channel voltage goes from V_S at the source end to V_D at the drain end. So the oxide has $\mathbf{V_G - V_S = 5\ V}$ **at the source end** and $\mathbf{V_G - V_D = 5\ V - 2\ V = 3\ V}$ **at the drain end** (L10-29). (At $V_D = 0$ V both ends would have 5 V, and the channel would be uniform.)

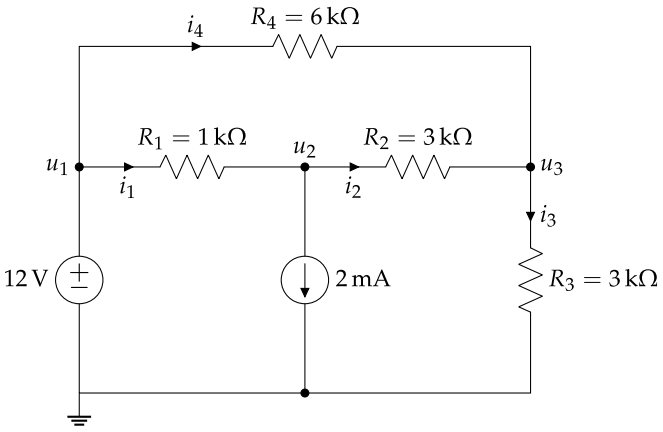
(**b**) Source end: 5 V − 1 V = 4 V. Drain end: 3 V − 1 V = 2 V. **Ratio = 2/4 = 1/2.** The channel near the drain holds half as many electrons per unit area. That part has a lower σ and more resistance, so I_D rises more slowly as V_D increases.

(**c**) Pinch-off occurs when the oxide voltage at the drain end falls to V_T: $V_G - V_D = V_T$, so $\mathbf{V_D = V_G - V_T = 4\ V}$ (this is $V_{DS,sat}$, L10-30). **Above this value, the channel from the source to the pinch-off point always has $\mathbf{V_{GS} - V_T = 4\ V}$ across it, so it carries the same current, $\mathbf{I_{D,sat}}$. The extra drain voltage drops across the short pinched-off region, where a strong field sweeps the electrons into the drain** (L10-29). I_D rises only a little, because the pinch-off point moves a small distance ΔL toward the source (L10-31).

Part II · Long problems (50 points)

Problem 9 CIRCUITS HARD 13 POINTS

Use nodal analysis for this circuit. The bottom node is the reference node (ground). The figure defines the node voltages u_1, u_2 and u_3 and the currents i_1 to i_4.



(a) Which node voltage do you know without a calculation? Give its value and the reason. (1 point)

(b) Write KCL at node u_2 and at node u_3. Use Ohm's law to write each resistor current in terms of the node voltages. (4 points)

(c) Solve for u_2 and u_3. (3 points)

(d) Find i_1 and i_4. Find the current that the 12 V source delivers. (2 points)

(e) Find the power of the 12 V source and of the 2 mA source. State whether each source absorbs or supplies power. The four resistors absorb 30 mW in total. Show that the powers balance. (3 points)

SOLUTION

(a) The 12 V source connects u_1 to the reference node, so $\mathbf{u_1 = 12\ V}$ (L6-18). The unknowns are u_2 and u_3.

(b) Work in kΩ, V and mA. Ohm's law gives each current in its arrow direction (L6-20): take the voltage at the arrow tail, subtract the voltage at the arrow head, and divide by R. So $i_1 = (12 - u_2)/1$, $i_2 = (u_2 - u_3)/3$, $i_3 = u_3/3$ and $i_4 = (12 - u_3)/6$. KCL at u_2: i_1 enters, and i_2 and the 2 mA source current leave, so $i_1 = i_2 + 2$ and $\mathbf{12 - u_2 = (u_2 - u_3)/3 + 2}$. Multiply by 3: $36 - 3{\cdot}u_2 = u_2 - u_3 + 6$, so $\mathbf{4{\cdot}u_2 - u_3 = 30}$. KCL at u_3: i_2 and i_4 enter, and i_3 leaves, so $i_2 + i_4 = i_3$ and $\mathbf{(u_2 - u_3)/3 + (12 - u_3)/6 = u_3/3}$. Multiply by 6: $2{\cdot}u_2 - 2{\cdot}u_3 + 12 - u_3 = 2{\cdot}u_3$, so $\mathbf{2{\cdot}u_2 - 5{\cdot}u_3 = -12}$.

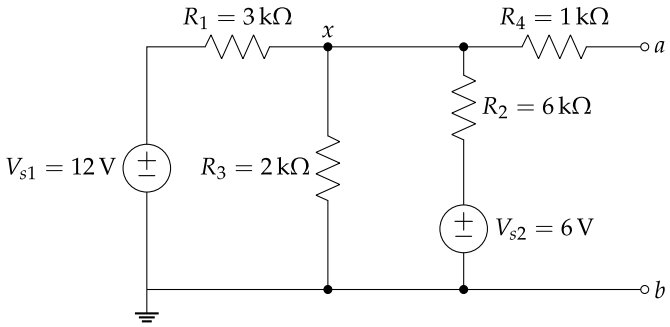
(c) Multiply the second equation by 2: $4{\cdot}u_2 - 10{\cdot}u_3 = -24$. Subtract it from the first equation: $9{\cdot}u_3 = 54$, so $\mathbf{u_3 = 6\ V}$. Then $4{\cdot}u_2 = 30 + 6 = 36$, so $\mathbf{u_2 = 9\ V}$. (Check in the second equation: $18 - 30 = -12$.)

(d) $\mathbf{i_1 = (12 - 9)/1 = 3\ mA}$ and $\mathbf{i_4 = (12 - 6)/6 = 1\ mA}$. Both values are positive, so both currents flow in the direction of their arrows. KCL at u_1: the 12 V source delivers $\mathbf{i_1 + i_4 = 4\ mA}$. (Not asked: $i_2 = (9 - 6)/3 = 1$ mA and $i_3 = 6/3 = 2$ mA. KCL check at u_3: 1 mA + 1 mA in = 2 mA out.)

(e) The 4 mA leaves the "+" terminal of the 12 V source, so the **12 V source supplies 12 V × 4 mA = 48 mW**. The voltage across the 2 mA source is $u_2 - 0 = 9$ V, with "+" at the top. Its current flows down through it and enters at the "+" end, so the **2 mA source absorbs 9 V × 2 mA = 18 mW**. Balance: the resistors absorb 30 mW (given; 9 + 3 + 12 + 6 mW), and 30 mW + 18 mW = 48 mW, which equals the power supplied. A current source can absorb power, as a voltage source can.

Problem 10 CIRCUITS HARD 12 POINTS

Two voltage sources drive the terminals a and b.



(a) Find the Thevenin voltage $V_{TH} = V_{ab}$ with a–b open. (3 points)

(b) Find the Thevenin resistance R_{TH}. (2 points)

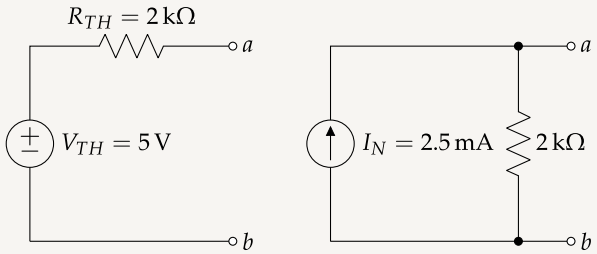
(c) Connect a wire from a to b. Find the Norton current I_N in this wire, and check that $I_N = V_{TH}/R_{TH}$. Then sketch the Thevenin and the Norton equivalent circuits (the sketch is worth 1 of the 3 points). (3 points)

(d) Find the reading of an ideal voltmeter connected between a and b. Then find the reading of a real voltmeter with $R_{in} = 18\,\text{k}\Omega$ between a and b. (2 points)

(e) A load $R_L = 8\,\text{k}\Omega$ is connected across a–b. Find the load current and the load voltage. (2 points)

SOLUTION

The equivalent circuits from (c):



(a) With a–b open, R_4 carries no current, so $V_{ab} = V_x$. KCL at node x (in kΩ, V and mA), with the currents in from the two source branches equal to the current out through R_3: $(12 - V_x)/3 + (6 - V_x)/6 = V_x/2$. Multiply by 6: $24 - 2{\cdot}V_x + 6 - V_x = 3{\cdot}V_x$, so $6{\cdot}V_x = 30$ and **$V_{TH} = V_x = 5$ V**.

(b) Replace both voltage sources by short circuits (L6-32). Then R_1, R_2 and R_3 all connect x to ground: $1/(1/3 + 1/6 + 1/2)\ \text{k}\Omega = 1/(6/6)\ \text{k}\Omega = 1\ \text{k}\Omega$. R_4 is in series: **$R_{TH} = 1\,\text{k}\Omega + 1\,\text{k}\Omega = 2\,\text{k}\Omega$**.

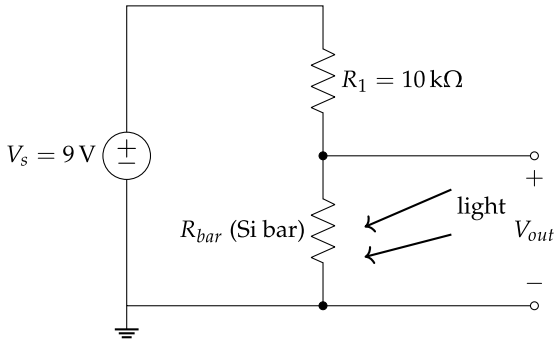
(c) The wire from a to b puts R_4 between x and ground. KCL at x: $(12 - V_x)/3 + (6 - V_x)/6 = V_x/2 + V_x/1$. Multiply by 6: $30 - 3{\cdot}V_x = 9{\cdot}V_x$, so $V_x = 2.5$ V. **$I_N = 2.5\ \text{V} / 1\,\text{k}\Omega = 2.5$ mA.** Check: $V_{TH}/R_{TH} = 5\ \text{V} / 2\ \text{k}\Omega = 2.5$ mA. **Thevenin equivalent: 5 V in series with 2 kΩ. Norton equivalent: 2.5 mA in parallel with 2 kΩ** (L6-33).

(d) An ideal voltmeter ($R_{in} \to \infty$) draws no current, so it **reads $V_{TH} = 5$ V**. The real voltmeter forms a divider with R_{TH}: **reading = 5 V × 18/(2 + 18) = 4.5 V**, which is 10 % low.

(e) **$I = 5\ \text{V} / (2\,\text{k}\Omega + 8\,\text{k}\Omega) = 0.5$ mA** and **$V = 0.5$ mA × 8 kΩ = 4 V**.

Problem 11 DEVICES HARD 13 POINTS

A bar of lightly doped n-type Si is a light sensor in a voltage divider. The bar has $N_D = 10^{14}$ cm^{-3}, a length of 2 mm and a cross-section of 1 mm × 0.1 mm. Use $\mu_n = 1250$ cm^2/V·s and $\mu_p = 500$ cm^2/V·s. The divider has $V_s = 9$ V and $R_1 = 10$ kΩ. V_{out} is the voltage across the bar.



(a) In the dark, find n and p in the bar. (2 points)

(b) In the dark, find σ, ρ and the bar resistance R_{bar}. Explain why you can ignore the holes. (3 points)

(c) Find V_{out} in the dark. (2 points)

(d) Red light with λ = 620 nm shines on the bar. Show that Si absorbs this light. The light creates $\Delta n = \Delta p = 5 \times 10^{14}$ cm^{-3} in the whole bar. Find the new σ. Use the ratio of the new σ to the dark σ to find the new R_{bar} and V_{out}. (4 points)

(e) Infrared light with λ = 1550 nm replaces the red light. Find V_{out}. Explain. (2 points)

SOLUTION

(a) Each donor gives one free electron: $\mathbf{n \approx N_D = 10^{14}\ cm^{-3}}$. $\mathbf{p = n_i^2/n = 10^{20}/10^{14} = 10^6\ cm^{-3}}$.

(b) $\sigma = q\cdot(\mu_n\cdot n + \mu_p\cdot p)$ (L9-19). The hole term $\mu_p\cdot p = 500 \times 10^6$ is 2.5×10^8 times smaller than the electron term $\mu_n\cdot n = 1250 \times 10^{14}$, so ignore it. $\mathbf{\sigma = 1.6 \times 10^{-19} \times 1250 \times 10^{14}\ S/cm = 0.02\ S/cm}$ ($1.6 \times 1250 = 1600 + 400 = 2000$) and $\mathbf{\rho = 1/\sigma = 50\ \Omega\cdot cm}$. L = 0.2 cm and A = 0.1 cm × 0.01 cm $= 10^{-3}$ cm^2, so L/A = 200 cm^{-1}. $\mathbf{R_{bar} = \rho L/A = 50\ \Omega\cdot cm \times 200\ cm^{-1} = 10^4\ \Omega = 10\ k\Omega}$.

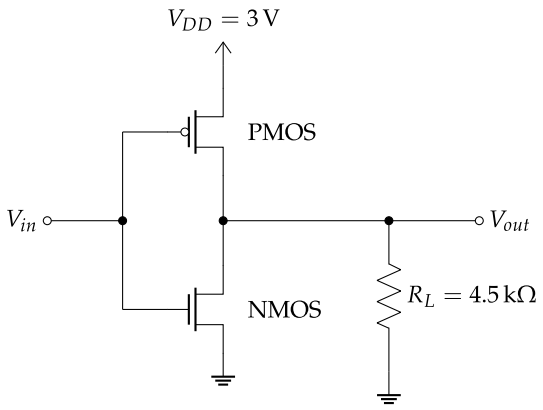
(c) Voltage divider (L5-12): $\mathbf{V_{out} = 9\ V \times 10/(10 + 10) = 4.5\ V}$.

(d) **E = 1240/620 = 2.0 eV, which is more than E_g(Si) = 1.12 eV, so Si absorbs the light** (L9-16). Each absorbed photon makes one electron–hole pair. Now $n = 10^{14} + 5 \times 10^{14} = 6 \times 10^{14}$ cm^{-3} and $p \approx 5 \times 10^{14}$ cm^{-3}, so the holes now matter. $\mathbf{\sigma = 1.6 \times 10^{-19} \times (1250 \times 6 \times 10^{14} + 500 \times 5 \times 10^{14})\ S/cm = 1.6 \times 10^{-19} \times (7.5 \times 10^{17} + 2.5 \times 10^{17})\ S/cm = 1.6 \times 10^{-19} \times 10^{18}\ S/cm = 0.16\ S/cm.}$ The light makes σ 8 times larger than in the dark (0.16/0.02 = 8). $R_{bar} = \rho L/A = L/(\sigma\cdot A)$ is proportional to $1/\sigma$, so R_{bar} falls by the same factor: $\mathbf{R_{bar} = 10\ k\Omega/8 = 1.25\ k\Omega}$. (Direct route: $\rho = 1/\sigma = 1/(0.16\ S/cm) = (100/16)\ \Omega\cdot cm = 6.25\ \Omega\cdot cm$, and $6.25\ \Omega\cdot cm \times 200\ cm^{-1} = 1250\ \Omega$.) Now $R_1 = 10\ k\Omega = 8 \times R_{bar}$, so $R_1 + R_{bar} = 9 \times R_{bar}$ and $\mathbf{V_{out} = 9\ V \times R_{bar}/(R_1 + R_{bar}) = 9\ V \times 1/9 = 1.0\ V}$. The light lowers V_{out} from 4.5 V to 1.0 V, so the circuit senses light.

(e) **E = 1240/1550 eV is less than 1 eV, because 1550 nm is longer than 1240 nm.** (The exact value is 0.8 eV.) **So E is below E_g(Si) = 1.12 eV.** Si does not absorb this light, so the light makes no electron–hole pairs. **R_{bar} stays at 10 kΩ, and V_{out} stays at 4.5 V.**

Problem 12 DEVICES HARD 12 POINTS

A CMOS inverter uses $V_{DD} = 3$ V. The NMOS has $k'_n = \mu_n C_{ox}(W/L) = 2$ mA/V^2 and $V_T = 1$ V. The PMOS has the same W/L, so $k'_p = 1$ mA/V^2, because holes have about half the mobility of electrons. The PMOS has $V_T = -1$ V. A transistor that is on, with a small $|V_{DS}|$, acts as a resistor $R_{CH} = 1/[k' \cdot (|V_{GS}| - |V_T|)]$.



(a) For $V_{in} = 0$ V and for $V_{in} = 3$ V, state which transistor is on. Find its R_{CH}. (3 points)

(b) The load $R_L = 4.5$ kΩ connects the output to GND. For $V_{in} = 0$ V, find V_{out} and the current from V_{DD}. For $V_{in} = 3$ V, find V_{out} and the current from V_{DD}. (4 points)

(c) By what factor must the PMOS W/L change to make its R_{CH} equal to the R_{CH} of the NMOS? (1 point)

(d) Remove R_L. The output now drives only the capacitance C = 20 fF of the next gates. The input falls from 3 V to 0 V, and the output rises from 0 V to 3 V. Find the charge that flows from V_{DD} onto C. The output makes 5×10^8 of these 0-to-1 transitions each second. Find the average current from V_{DD} and the average power $V_{DD} \cdot I$. (4 points)

SOLUTION

(a) **$V_{in} = 0$ V: the PMOS is on** ($V_{GS} = 0 - 3$ V $= -3$ V, which is below -1 V), and the NMOS is off. **$R_{CH,p} = 1/[1\text{ mA/V}^2 \times (3\text{ V} - 1\text{ V})] = 1/(2 \times 10^{-3}\text{ A/V}) = 500\ \Omega$** (L10-33). **$V_{in} = 3$ V: the NMOS is on** ($V_{GS} = 3$ V, above 1 V), and the PMOS is off. **$R_{CH,n} = 1/[2\text{ mA/V}^2 \times 2\text{ V}] = 250\ \Omega$.**

(b) **$V_{in} = 0$ V:** the PMOS channel (500 Ω) and R_L form a voltage divider from V_{DD}. **$V_{out} = 3$ V $\times$ 4.5/(4.5 + 0.5) = 2.7 V. The current from V_{DD} is 3 V / 5 kΩ = 0.6 mA** (a power of 1.8 mW). The PMOS has only $V_{SD} = 0.3$ V across it, which is small compared with $V_{SG} - |V_T| = 2$ V, so the resistor model is good here. (Check, not required: the full linear-region equation has the extra term $-\frac{1}{2} \cdot V_{SD}^2$. At $V_{SD} = 0.3$ V, this term lowers the PMOS current by the fraction $V_{SD}/[2 \cdot (V_{SG} - |V_T|)] = 0.3/4 = 7.5$ %. So the exact V_{out} is a little below 2.7 V.) **$V_{in} = 3$ V:** the NMOS connects the output to GND, so **$V_{out} = 0$ V**, and R_L carries no current. The PMOS is off, so **the current from V_{DD} is ≈ 0** (only the leakage current I_{off}, L10-34).

(c) R_{CH} is proportional to $1/k' = 1/[\mu_p C_{ox}(W/L)]$. To halve $R_{CH,p}$ from 500 Ω to 250 Ω, **double the PMOS W/L** (make the PMOS about 2 times wider).

(d) From C = Q/V (L6-34): **$Q = C \cdot V_{DD} = 20\text{ fF} \times 3\text{ V} = 60\text{ fC} = 6 \times 10^{-14}$ C** for each transition. Current is charge per unit time (i = dQ/dt), so **$I = 6 \times 10^{-14}\text{ C} \times 5 \times 10^8\text{ s}^{-1} = 3 \times 10^{-5}\text{ A} = 30\ \mu\text{A}$. $P = V_{DD} \cdot I = 3\text{ V} \times 30\ \mu\text{A} = 90\ \mu\text{W}$.** This power flows only while the gate switches, unlike the static power in (b).