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ECE 105: Introduction to Electrical Engineering

Lecture 21

Optical Communications

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How have we done optical communications in the past?

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People sent messages with light long before electricity: switch the light, agree on a code.



Smoke signals

Puffs of smoke seen from a distant hill



Signal lamp

Shutters flash Morse code between ships



Heliograph

A mirror on a tripod flashes sunlight



Fiber optics

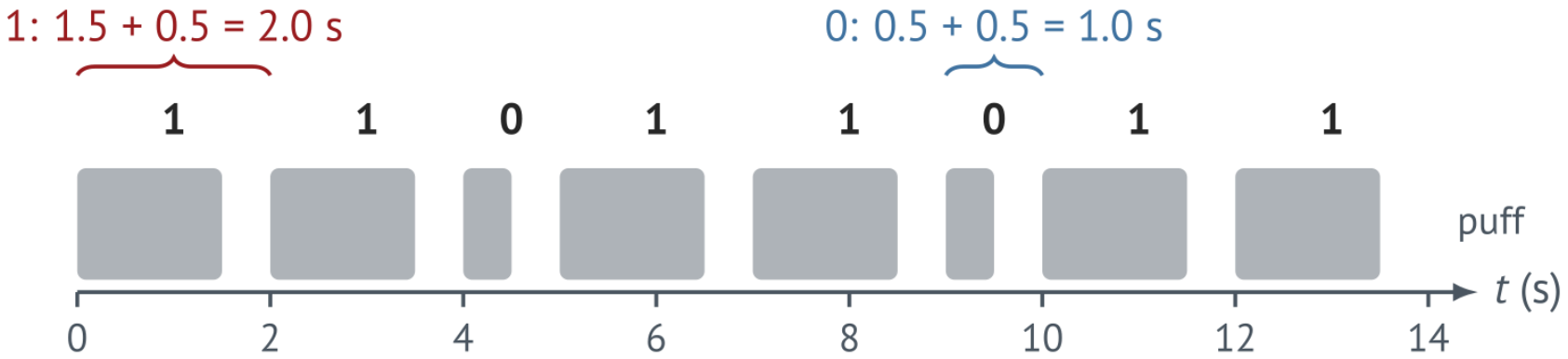
Light pulses in glass, Tbit/s today

Common idea: switch the light between states the receiver can tell apart, and agree in advance on what each state means.

How fast could we communicate using smoke signals?

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Send a 1 as a long puff and a 0 as a short puff; the puffs and the gaps set the bit rate.



EXAMPLE

Short puff 0.5 s, long puff 1.5 s, and at least 0.5 s of gap so the observer sees separate puffs. If 1s and 0s are equally likely, what is the average bit rate?

1. A 0 takes 0.5 + 0.5 = **1.0 s**
2. A 1 takes 1.5 + 0.5 = **2.0 s**
3. Average time per bit: (1.0 + 2.0)/2 = **1.5 s**
4. Bit rate: 1 bit / 1.5 s ≈ **0.67 bit/s**; one byte takes about 12 s

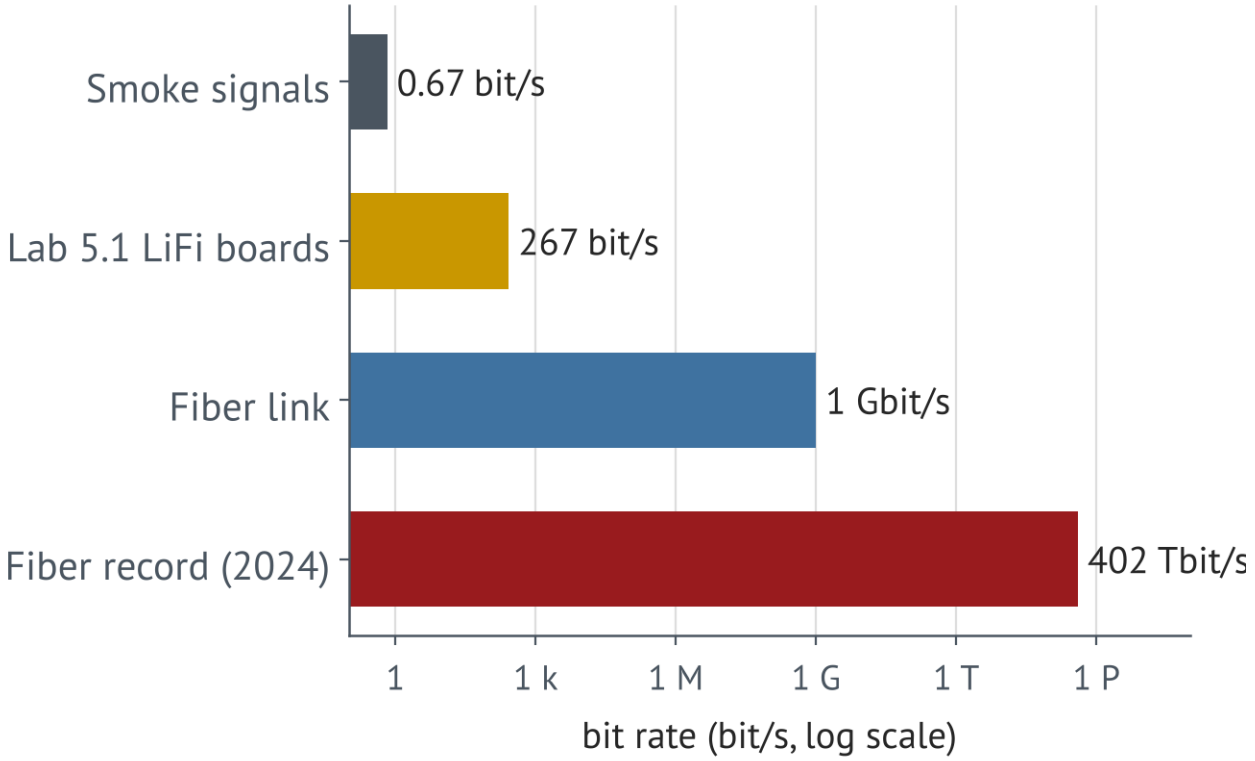
$$R_b = \frac{1}{\bar{T}_b} = \frac{1 \text{ bit}}{1.5 \text{ s}} \approx 0.67 \text{ bit/s}$$

Byte 11011011 above: six 1s and two 0s take 14 s

Smoke signals versus fiber

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One fiber channel at 1 Gbit/s sends about a billion times more bits per second than smoke.



Fiber versus smoke

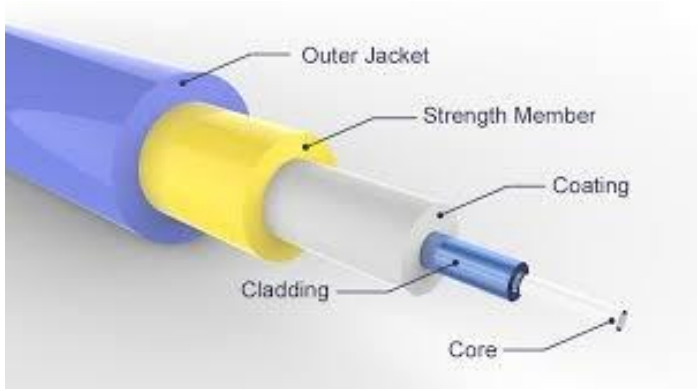
$$\frac{10^9 \text{ bit/s}}{0.67 \text{ bit/s}} \approx 1.5 \times 10^9$$

- An LED or laser switches in nanoseconds; a puff takes a second
- A photodiode, not a person, watches the light
- Many wavelengths share one fiber: the record used 1505 channels

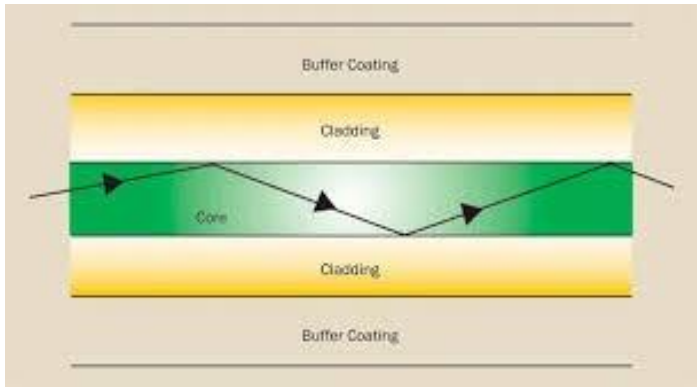
Lab 5.1 rate from the textbook (Task 1); fiber record: NICT, 2024.

Optical Fiber

A fiber guides light from one end to the other inside a glass core, with almost no loss.

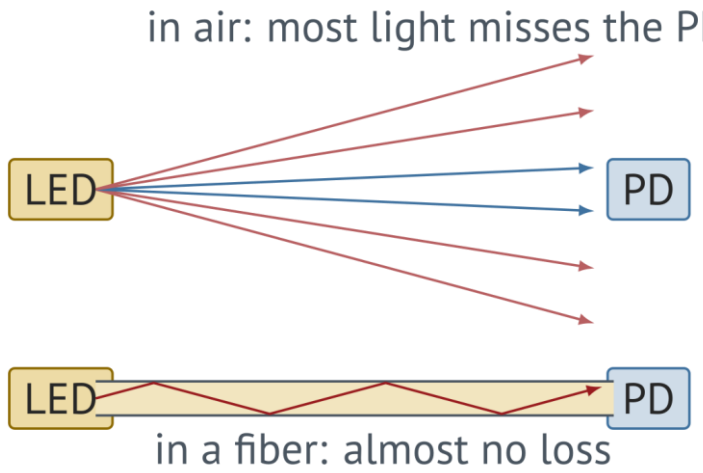


Cable layers: core, cladding, coating, strength member, jacket



Light zigzags along the core, reflected at the cladding

- **Core:** silica glass, n ≈ 1.47
- **Cladding:** glass, slightly lower n
- Light meets the wall at a grazing angle and is **fully reflected**: total internal reflection
- Coating, strength member and jacket protect the thin glass

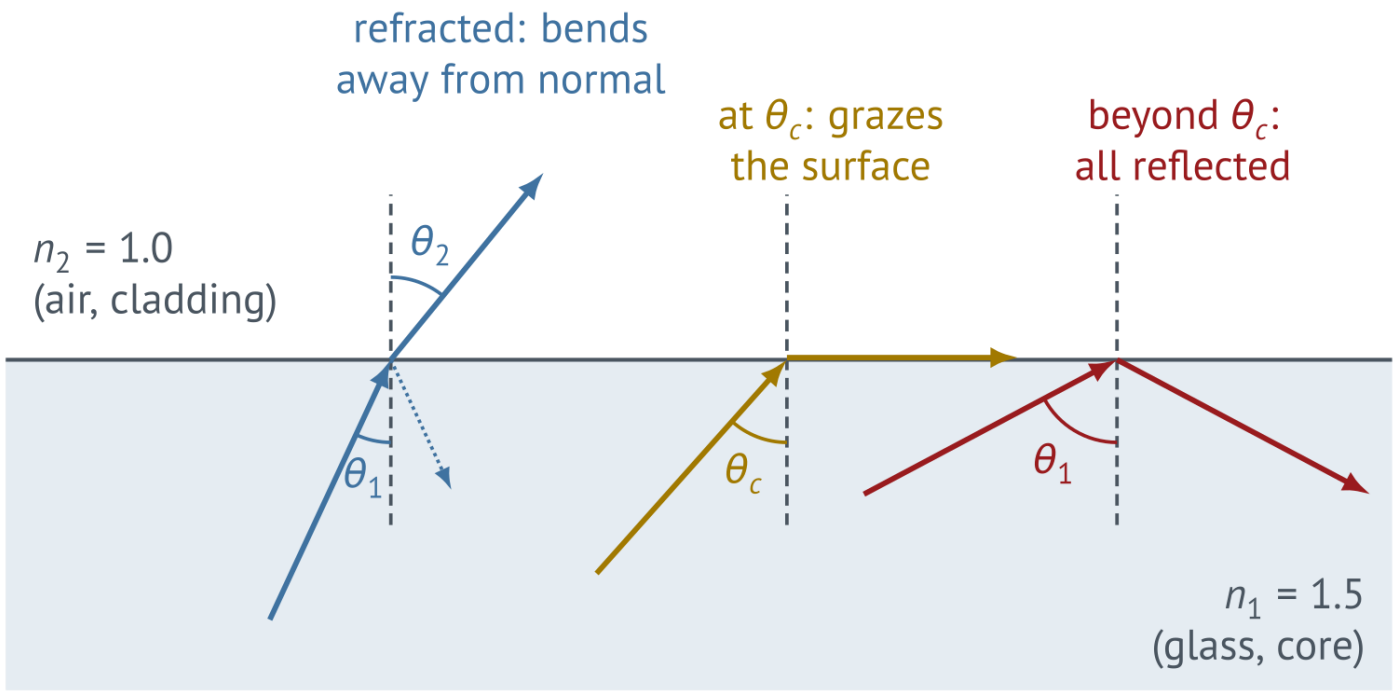


Without the fiber, light spreads and most never reaches the detector.

Total internal reflection

From high n to low n, light bends away from the normal; past the critical angle it is all reflected.

$$n = \frac{c}{v}$$ **refractive index**



Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Critical angle (needs $n_1 > n_2$)

$$\theta_c = \arcsin\frac{n_2}{n_1}$$

- Angles are measured from the **normal**
- $\theta_1 > \theta_c$: no light escapes
- Core n_1 > cladding n_2 keeps light in the fiber

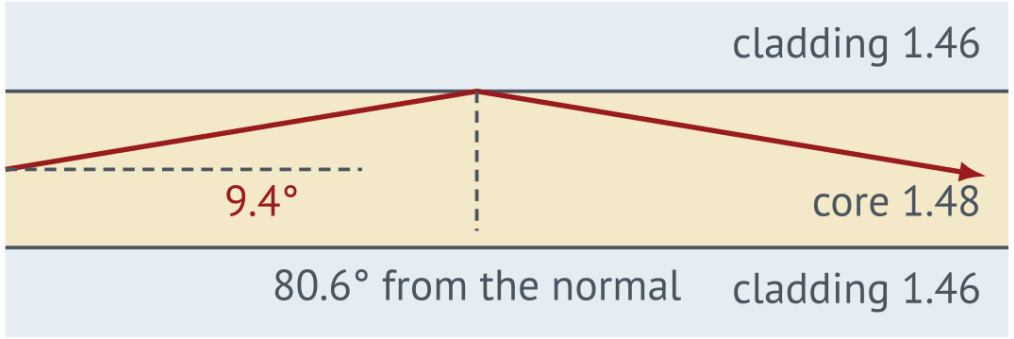
Total internal reflection: critical angle

The closer the two indices, the larger θc; a fiber traps only rays near its axis.

EXAMPLE

Find θ_c for (a) water into air, (b) n = 3 into n = 2, (c) a fiber core (1.48) into its cladding (1.46).

1. Water (1.333) into air: $\theta_c = \arcsin(1/1.333) =$ **48.6°**
2. n = 3 into n = 2: $\theta_c = \arcsin(2/3) =$ **41.8°**
3. Fiber: $\theta_c = \arcsin(1.46/1.48) =$ **80.6°**
4. Rays within 90° − 80.6° = **9.4°** of the axis stay trapped
5. Speed in the core: $c/1.48 \approx 2.0 \times 10^8$ m/s



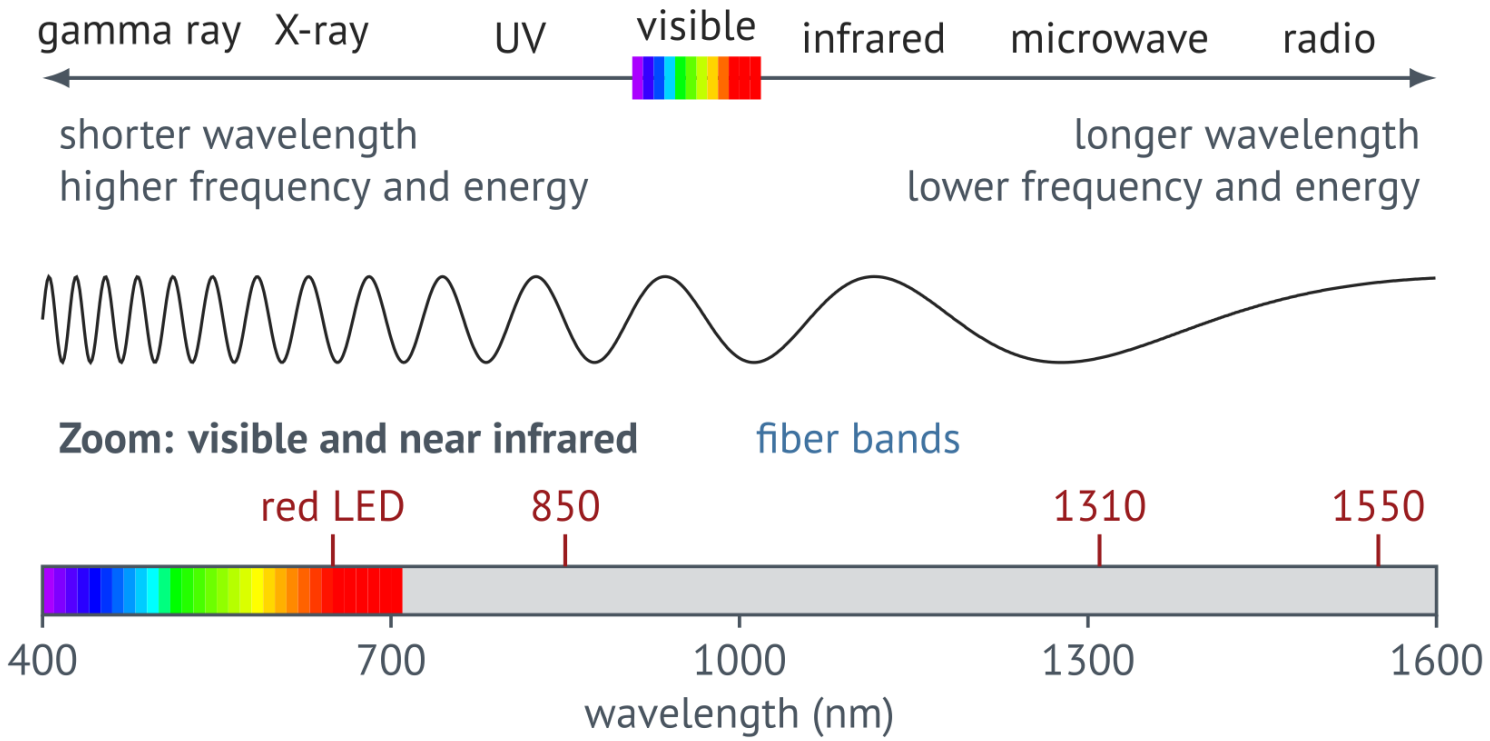
Critical angle

$$\theta_c = \arcsin\frac{n_2}{n_1} = \arcsin\frac{1.46}{1.48} = 80.6^\circ$$

Small index step, large θ_c

General properties of light

Light is an electromagnetic wave; its wavelength sets the color, and c = λf links it to frequency.



Wave relation

$$c = \lambda f$$

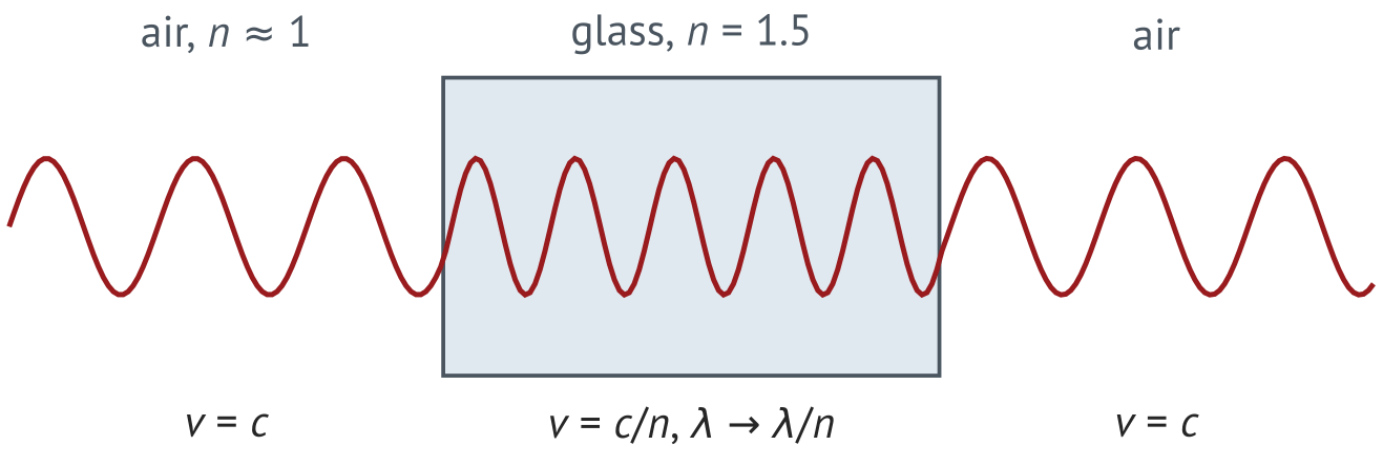
- λ: the length over which the wave repeats
- Visible: 400 to 700 nm
- Fiber: near infrared, 850 to 1550 nm

650 nm (red): $f \approx 4.6 \times 10^{14}$ Hz

1550 nm: $f \approx 1.9 \times 10^{14}$ Hz

Speed of light in different mediums

Light slows down inside matter; the refractive index n = c/v says by how much.



Refractive index

$$n = \frac{c}{v}$$

- n: refractive index
- c = 3.00 × 10^8 m/s in vacuum
- v: speed in the medium

- Electrons in the material oscillate and form dipoles that re-radiate; the sum is a **slower** wave
- Same frequency, so the wavelength shrinks to λ/n

Most optical links: n ≈ 1.5

v = c/1.5 = 2.0 × 10^8 m/s, so 1 km of fiber takes 5 µs

Refractive index of different materials

n depends on the material and on the wavelength; most glasses and plastics sit near 1.5.

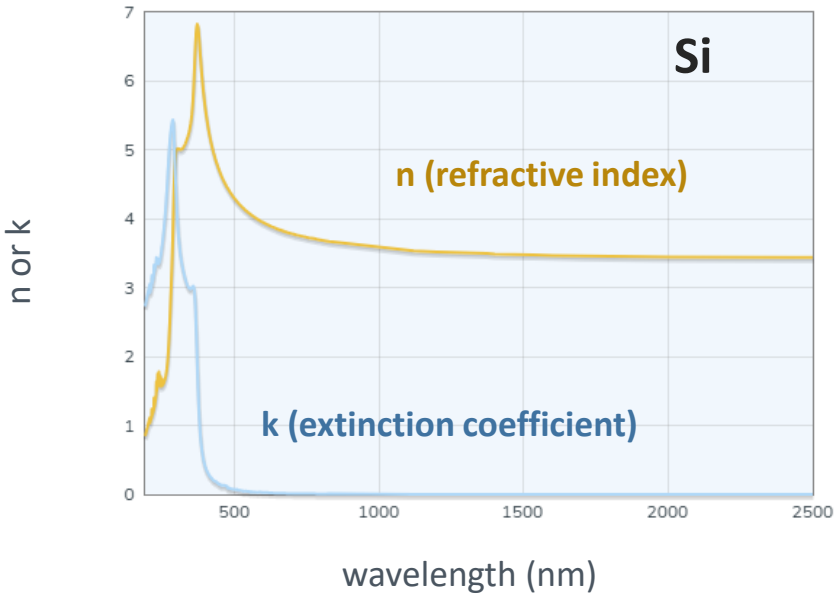
Gases and liquids	n
Vacuum	1 (exact)
Air (0 °C, 1 atm)	1.000293
Helium	1.000036
Hydrogen	1.000132
Carbon dioxide	1.00045
Water (20 °C)	1.333
Ethanol	1.36
Olive oil	1.47

Solids	n
Ice	1.31
Fused silica (quartz)	1.46
PMMA (acrylic)	1.49
Window glass	1.52
Polycarbonate	1.58
Flint glass	1.69
Sapphire	1.77
Cubic zirconia	2.15
Diamond	2.417
Moissanite	2.65

Values from Wikipedia, List of refractive indices.

Glasses and plastics: n ≈ 1.5

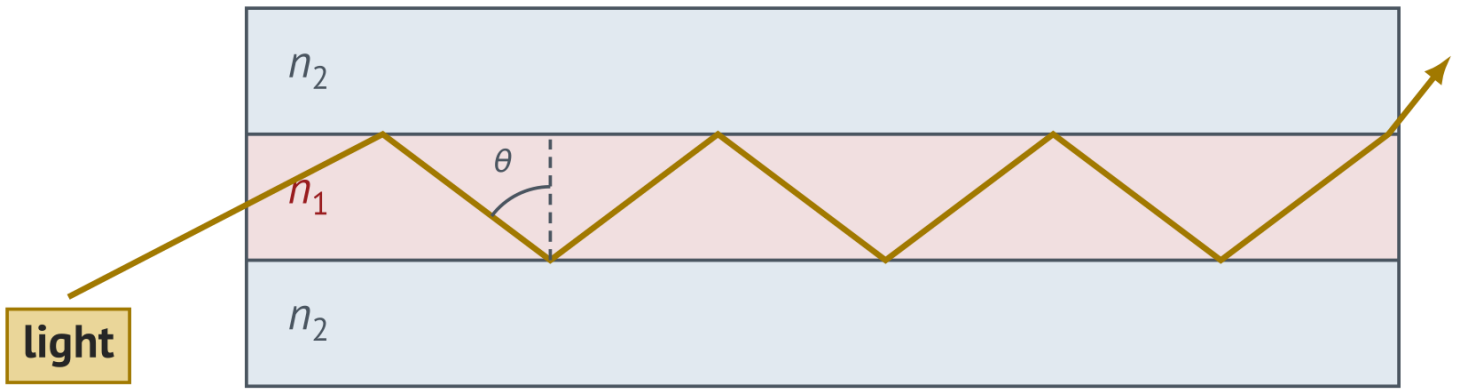
Gases: n ≈ 1



Silicon: n rises in the ultraviolet and settles near 3.5 in the infrared; k (absorption) drops to zero past about 1100 nm.

Waveguides

A high-index core between lower-index layers traps light by repeated total internal reflection.



Guiding condition

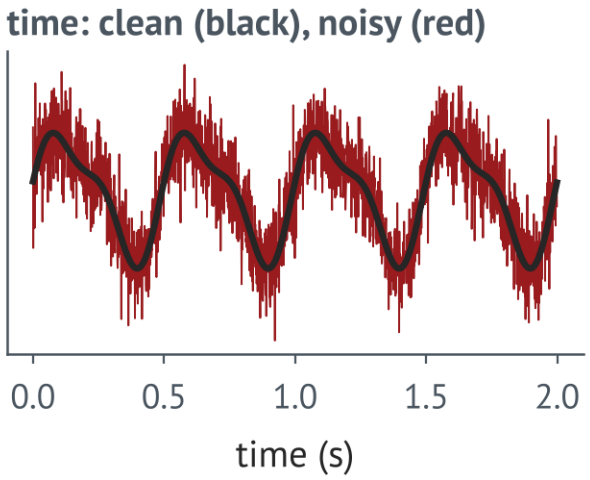
$$n_1 > n_2, \quad \theta > \theta_c$$

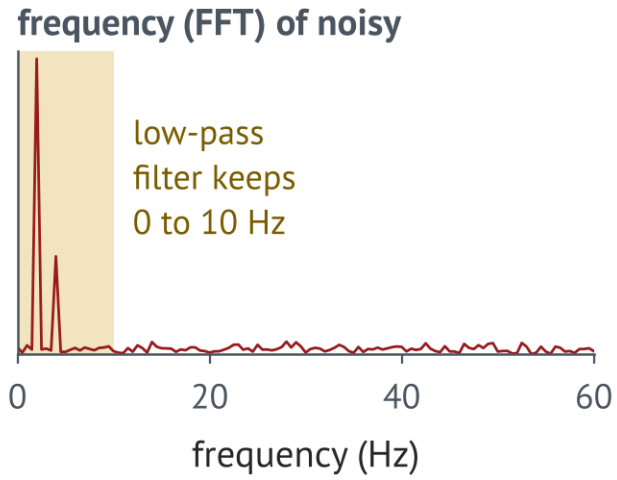
- Core: higher index n_1
- Cladding: lower index n_2
- Fiber: a cylindrical waveguide
- Photonic chips: silicon (n ≈ 3.5) in silica (n ≈ 1.45)

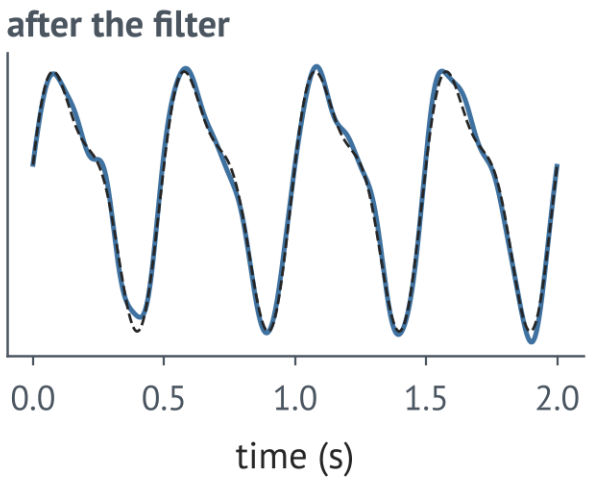
- Each bounce meets the wall at the same angle θ, so if one bounce is total, all of them are
- Light leaves at the far end, or where the guide bends too sharply

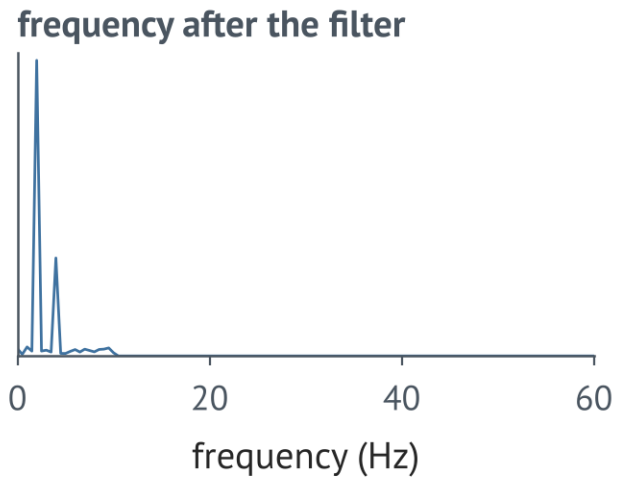
Noise

Noise is anything in your signal you did not want; the frequency view shows how to remove it.









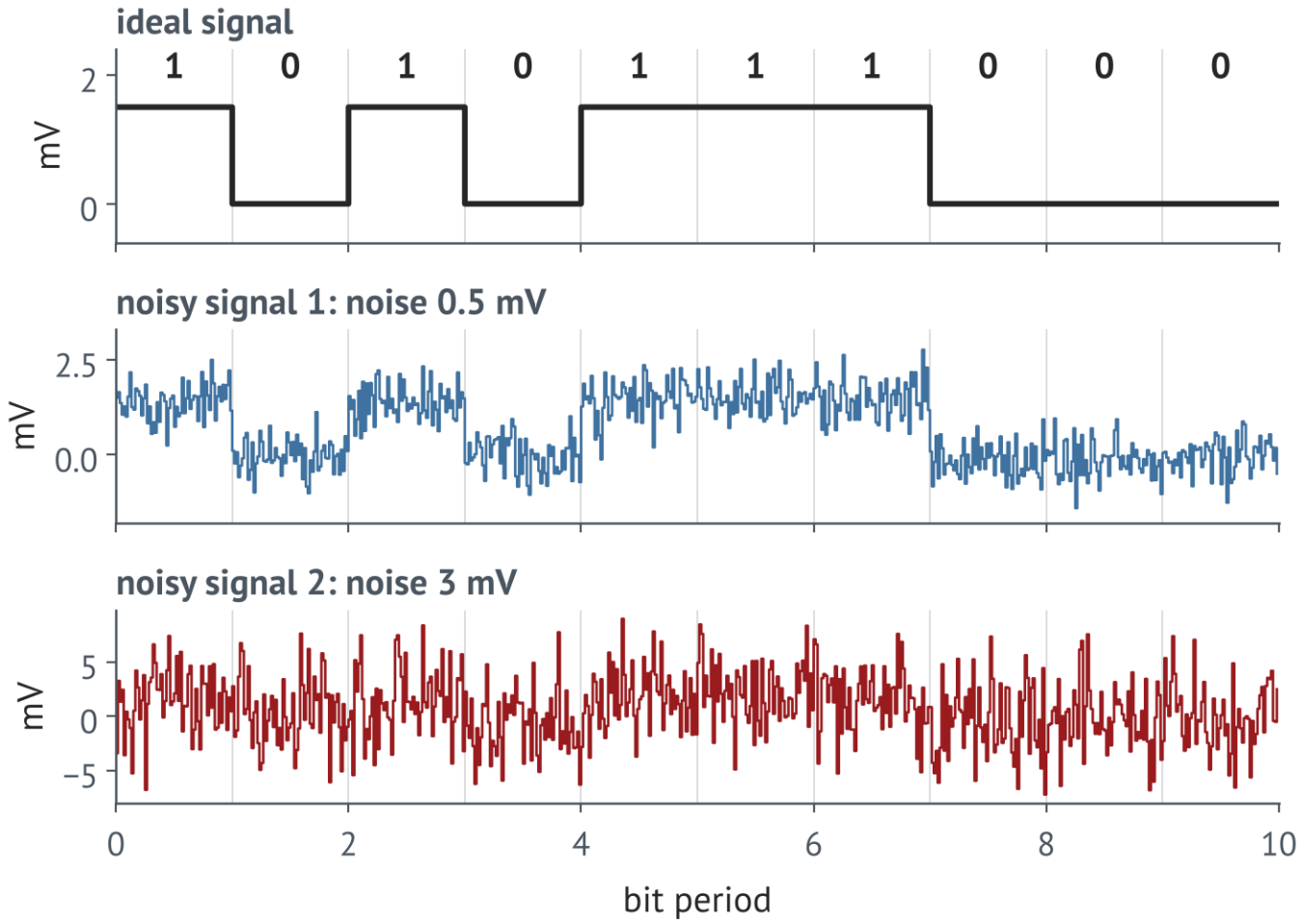
- Red: signal + noise; black: the clean signal
- The FFT shows the signal at **low frequency** and the noise spread over **all** frequencies
- A **low-pass filter** keeps the signal band and drops most of the noise

Narrow the band to what the signal needs: less noise gets through.

Signal to noise ratio

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SNR compares signal power with noise power; when noise matches the signal, the bits are lost.



Signal to noise ratio

$$\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{v_s^2}{v_n^2}$$

1. Signal 1: v_s/v_n = 1.5 mV / 0.5 mV = 3, so SNR = **9** (**9.5 dB**)
2. Signal 2: 1.5 mV / 3 mV = 0.5, so SNR = **0.25** (**−6.0 dB**): bits lost
3. In decibels: $\text{SNR}_{\text{dB}} = 20 \log_{10}(v_s/v_n)$

Optical Fiber Transmission Speeds

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Records send hundreds of wavelengths through one fiber, each channel at 100 to 400 Gbit/s.

Year	Organization	Aggregate speed	WDM channels	Per channel	Distance
2009	Alcatel-Lucent	15.5 Tbit/s	155	100 Gbit/s	7000 km
2010	NTT	69.1 Tbit/s	432	171 Gbit/s	240 km
2011	NEC	101.7 Tbit/s	370	273 Gbit/s	165 km
2011	KIT	26 Tbit/s	336	77 Gbit/s	50 km
2016	BT & Huawei	5.6 Tbit/s	28	200 Gbit/s	~140 km
2016	Nokia Bell Labs, Deutsche Telekom, TU Munich	1 Tbit/s	4	250 Gbit/s	419–951 km
2016	Nokia-Alcatel-Lucent	65 Tbit/s			6600 km
2017	BT & Huawei	11.2 Tbit/s	28	400 Gbit/s	250 km
2020	RMIT, Monash & Swinburne Universities	39.0–40.1 Tbit/s	160	244 Gbit/s	76.6 km
2020	UCL	178.08 Tbit/s	660 (S, C, L bands)	270 Gbit/s	40 km
2023	NICT	301 Tbit/s	1097 (E, S, C, L)	250–300 Gbit/s	50–150 km
2024	NICT	402 Tbit/s	1505 (O, E, S, C, L, U)	170–320 Gbit/s	50 km

Aggregate = channels × per channel: 402 Tbit/s ÷ 1505 ≈ 267 Gbit/s

Data: Wikipedia, fiber-optic transmission records.

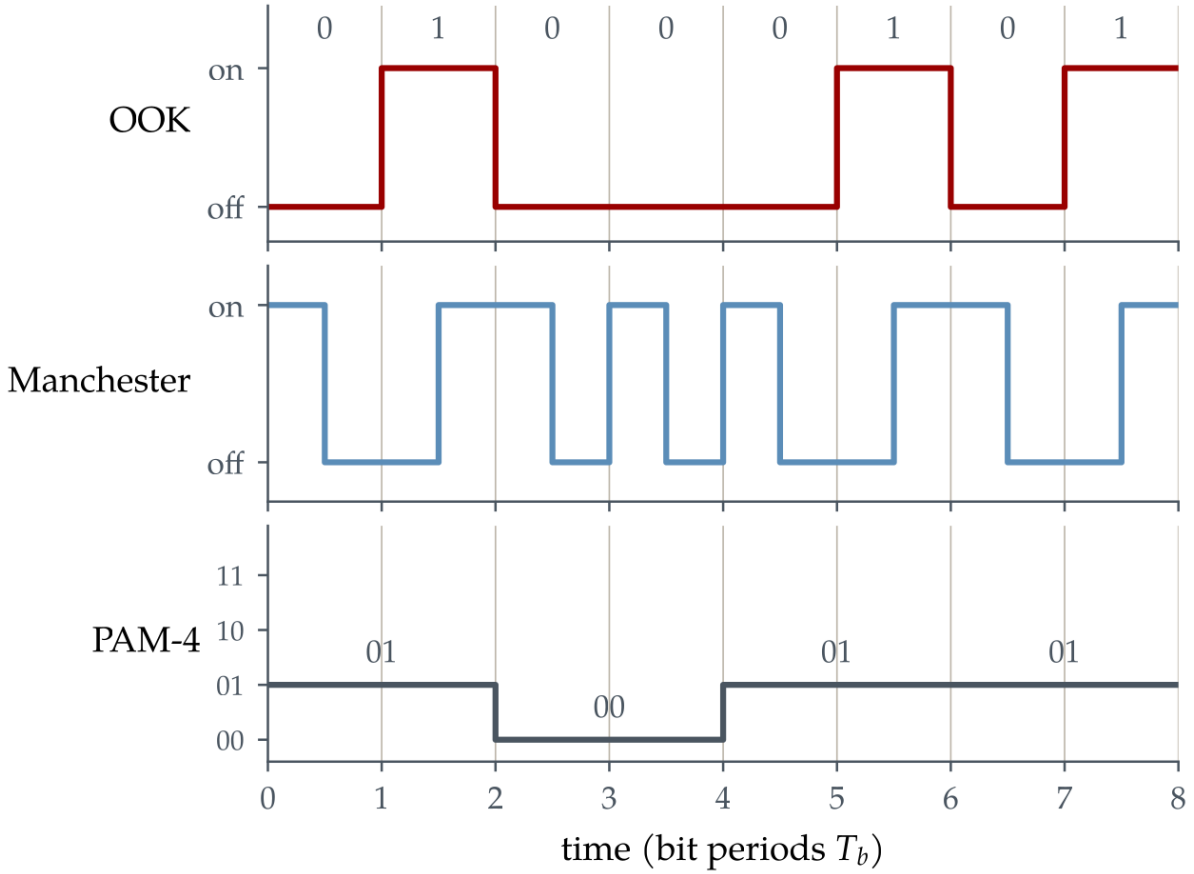
Optical Modulation Techniques

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Intensity modulation sends bits as brightness levels; on-off keying uses two: on for 1, off for 0.

- **Intensity modulation:** change the brightness; each level stands for a symbol
- **On-off keying (OOK):** two levels, LED on = 1 and off = 0
- **Manchester:** a transition in the middle of every bit keeps the clock aligned
- **PAM-4:** four levels, so each symbol carries 2 bits

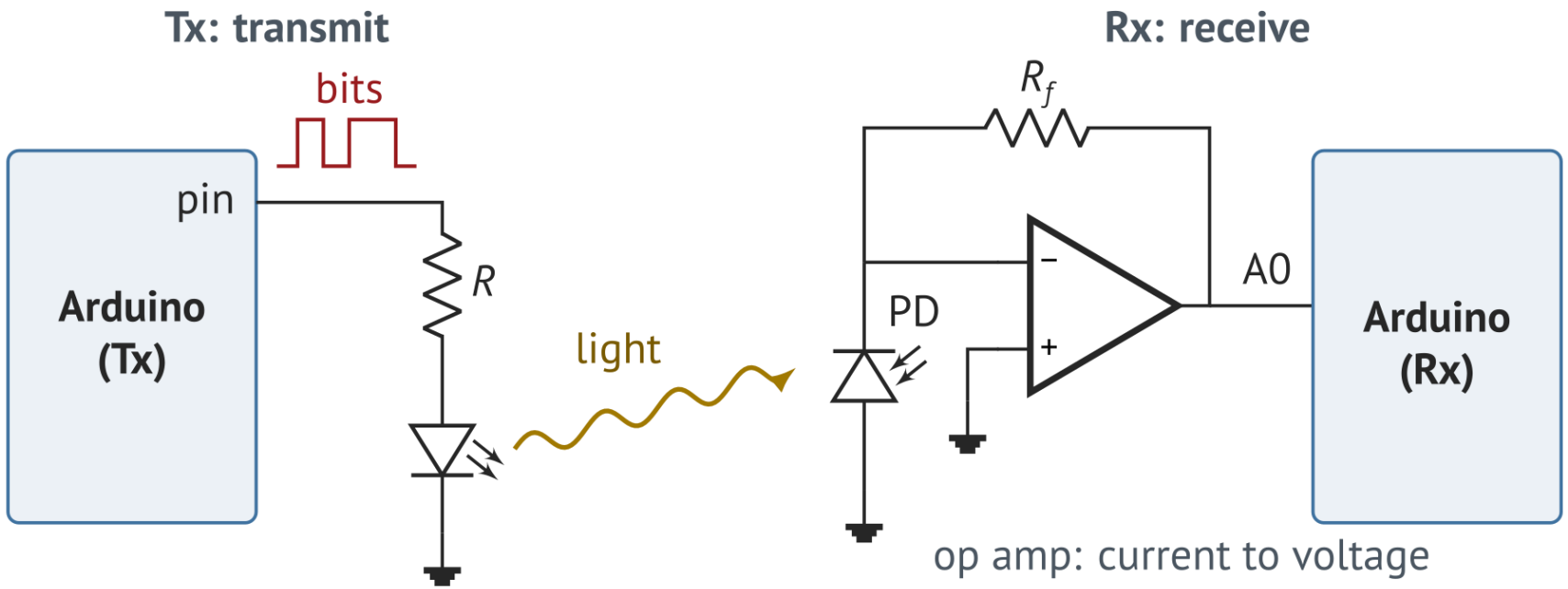
More levels: faster, but the levels sit closer together, so noise causes errors sooner.



The letter E (01000101) sent three ways. OOK: one level per bit. Manchester: two half-bits. PAM-4: one of four levels per pair of bits.

Let's design optical comm system

Transmit with a pin and an LED; receive with a photodiode, an op amp, and a second Arduino.



Transmitter

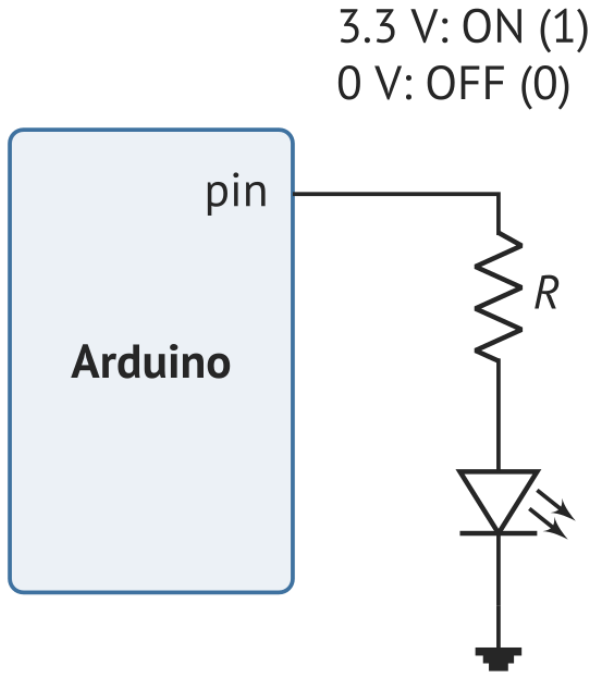
- A digital pin sends the bits: 1 = LED on, 0 = LED off
- The resistor limits the LED current

Receiver

- The photodiode gives a **current**; the analog input needs a **voltage**
- An op amp converts current to voltage and amplifies

Let's design optical comm system: transmitter

The pin sets 3.3 V (LED on, 1) or 0 V (LED off, 0); the delay in the code sets the bit rate.




```
const int LED_PIN = 2;           // any digital pin
void setup() {
  pinMode(LED_PIN, OUTPUT);
}
void loop() {
  digitalWrite(LED_PIN, HIGH);   // send 1: 3.3 V
  delay(1);                      // hold for 1 ms
  digitalWrite(LED_PIN, LOW);    // send 0: 0 V
  delay(1);
}
```

Bit rate set by the code

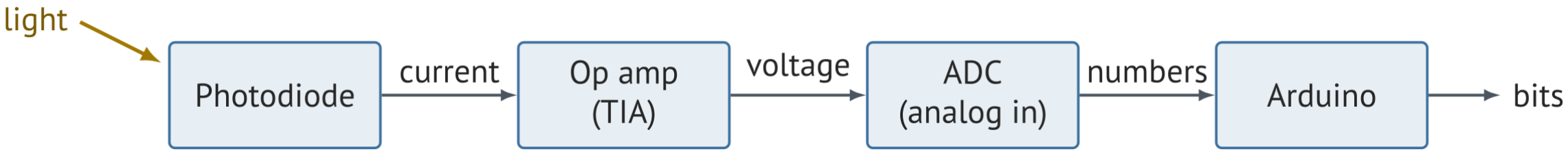
$$R_b = \frac{1}{T_b} = \frac{1}{1\ \text{ms}} = 1000\ \text{bit/s}$$

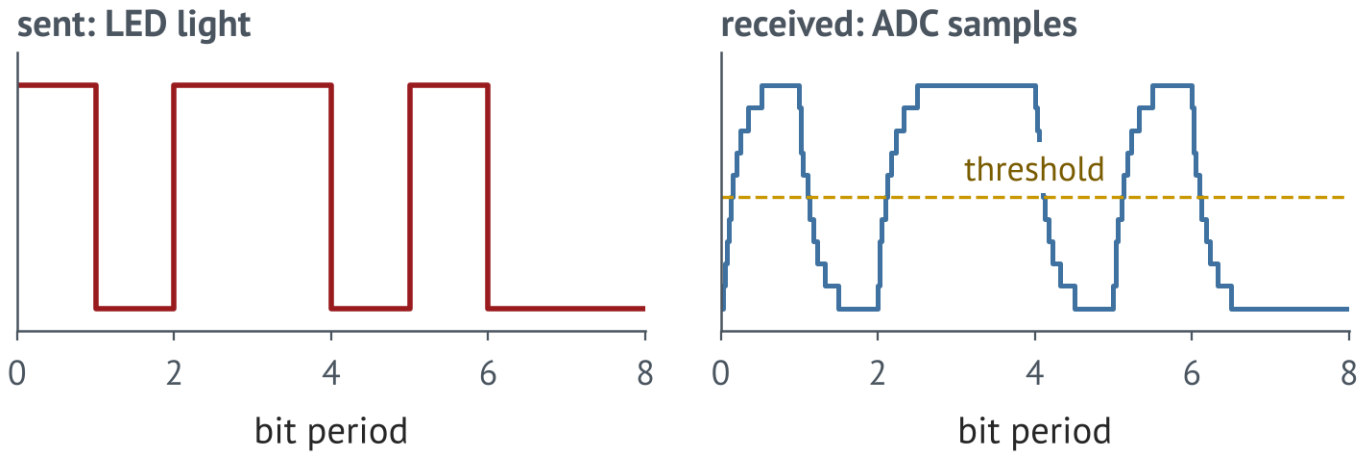
1. Hardware: 64 MHz clock, one cycle every 15.6 ns
2. Code: delay(1) holds each bit for 1 ms

How fast can you turn it on and off? The code, not the LED, is the limit.

Let's design optical comm system: receiver

The photodiode current becomes a voltage, then numbers; the Arduino decides 1 or 0.



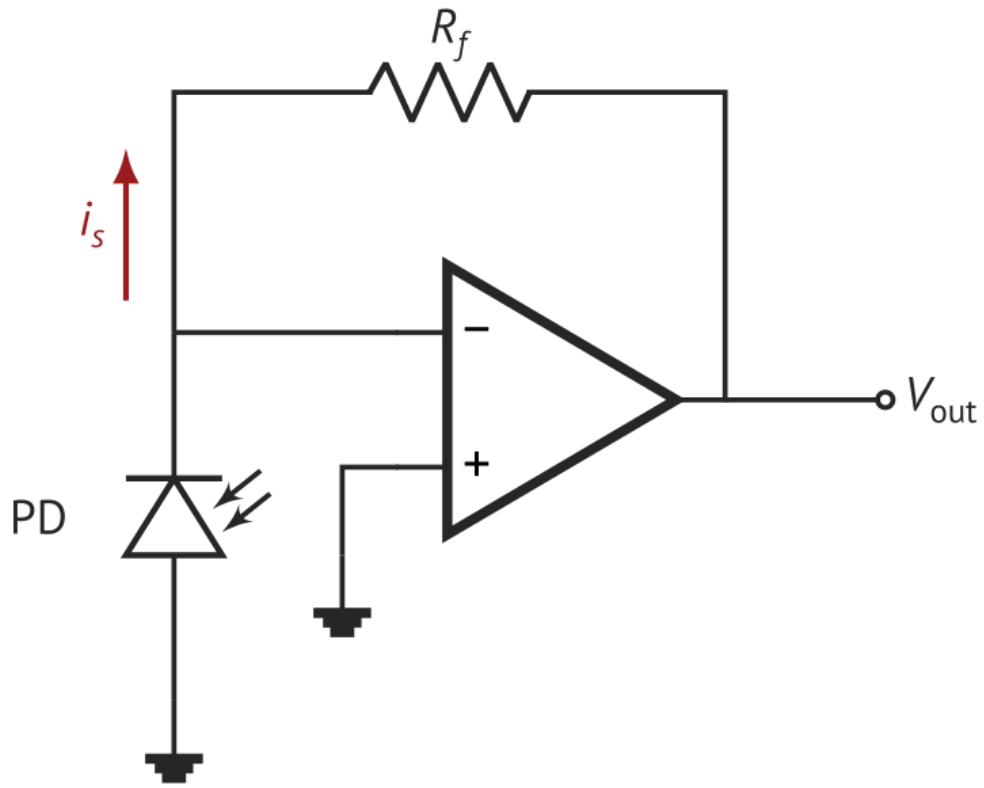


- Edges arrive **rounded**: the receiver has limited bandwidth
- Levels arrive in **steps**: the ADC has a finite number of levels
- Compare each sample with a threshold: 1 or 0

The photodiode gives an analog **current**; the Arduino's analog input needs a **voltage**: the op amp makes the conversion.

The transimpedance amplifier

The op amp holds its input at 0 V, so all the photocurrent flows through R_f: $V_{out} = -i_s R_f$.



PD: photodiode; R_f: feedback resistor

1. The op amp holds the − input at **0 V** (virtual ground)
2. No current enters the op amp, so i_s flows through R_f
3. KCL: $(0 - V_{out})/R_f = i_s$

$$V_{out} = -i_s R_f \qquad |\text{gain}| = R_f \ \text{(V/A)}$$

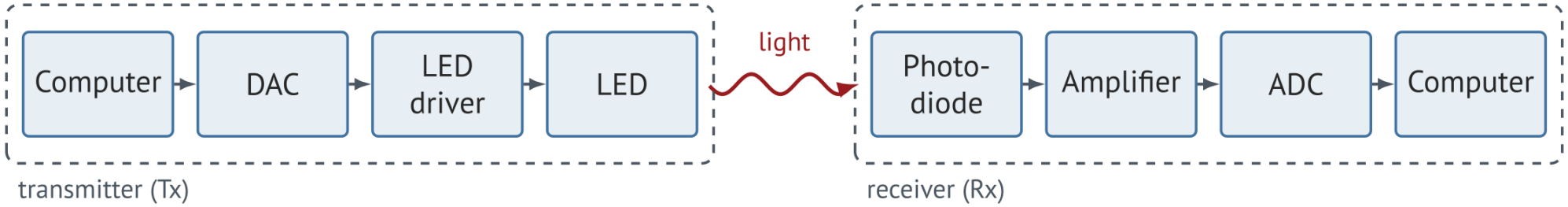
Gain examples

1 µA × 1 MΩ = **1 V**; 1 mA × 1 kΩ = **1 V**

$\mu = 10^{-6}$, $m = 10^{-3}$, $k = 10^{3}$, $M = 10^{6}$

Let's talk about a real world system

Numbers to voltage to light; then light to current to voltage to numbers again.



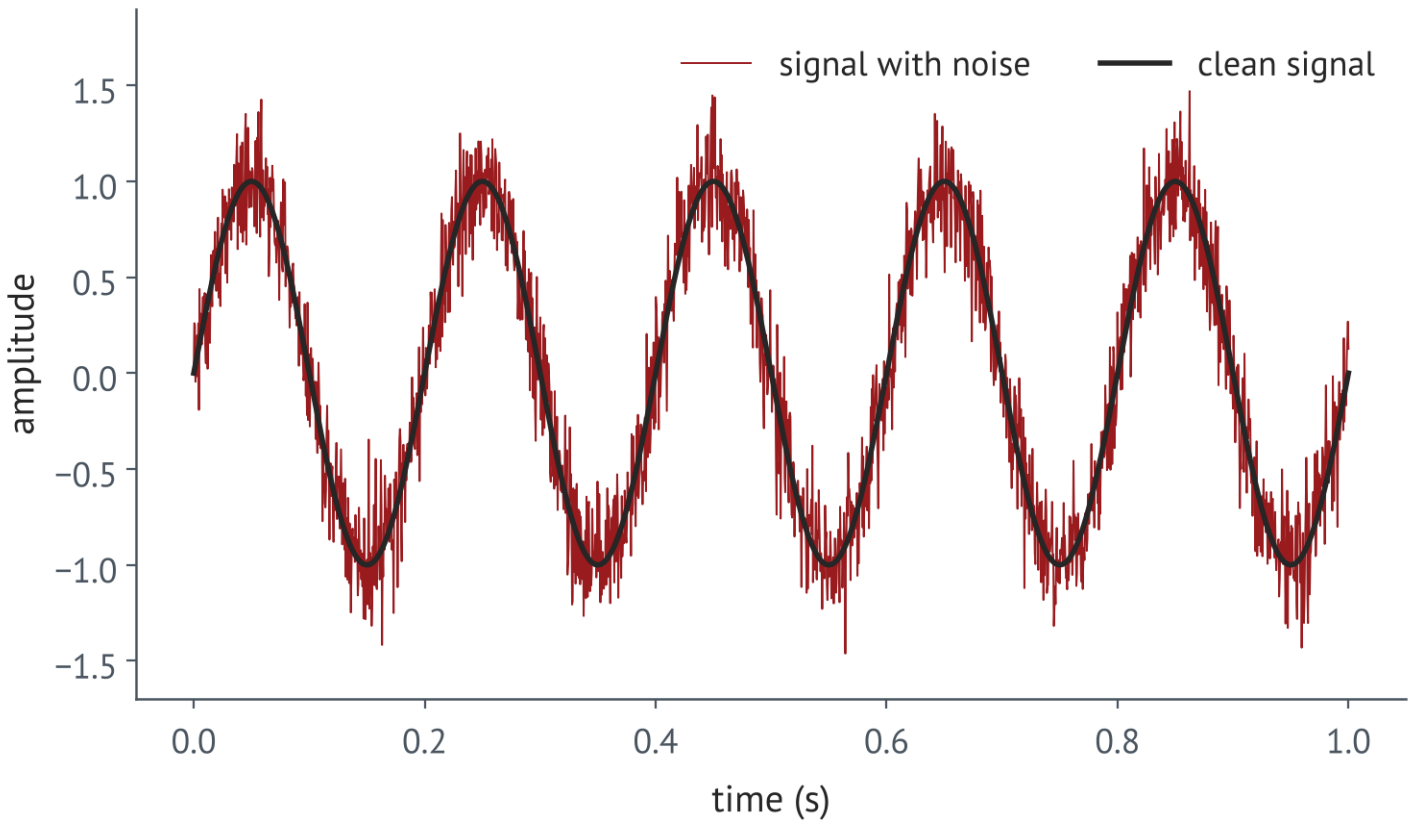
- Seems easy: flash an LED, detect the flashes, done
- But how much data can such a signal carry?
- What components do we need, and what does each add?

On the demo board

- Computer: the Arduino
- DAC: digital-to-analog converter (or a GPIO pin)
- ADC: analog-to-digital converter
- More signal: a brighter LED or more amplifier gain

What is noise?

Noise is anything in your signal that you did not want: random wiggles riding on the signal.



A 5 Hz sine wave with random noise added (simulation).

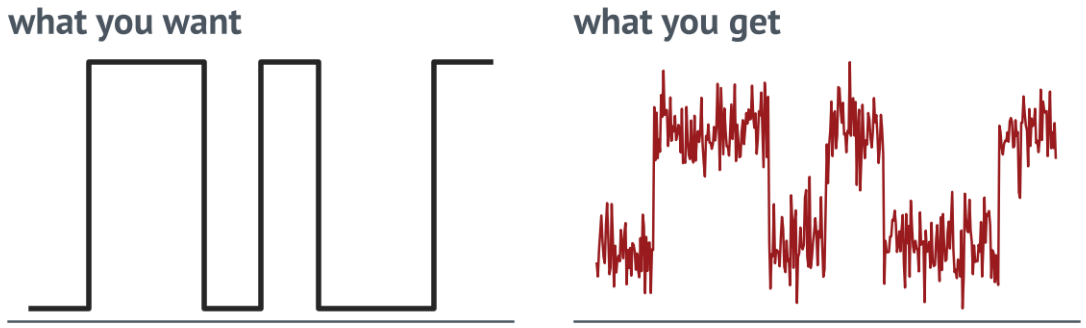
- **From outside:** ambient light, power-line hum, motors
- **From inside:** the physics of every resistor, diode and transistor
- Inside noise cannot be removed, only managed

In a link, noise can push a level across the threshold and turn a 1 into a 0.

Types of Noise

Three mechanisms live inside every component: heat, defects, and the graininess of charge.

- Noise: anything you do not want in your signal
- Many sources, many effects on your system



Thermal (Johnson-Nyquist)

$$v_n = \sqrt{4kTRB}$$

Random thermal motion of carriers in resistors

Grows with T, R and B

Flicker (1/f)

$$S(f) \propto 1/f^{\alpha}$$

Defects trap and release carriers

Dominates at low frequency

Shot

$$i_n = \sqrt{2qIB}$$

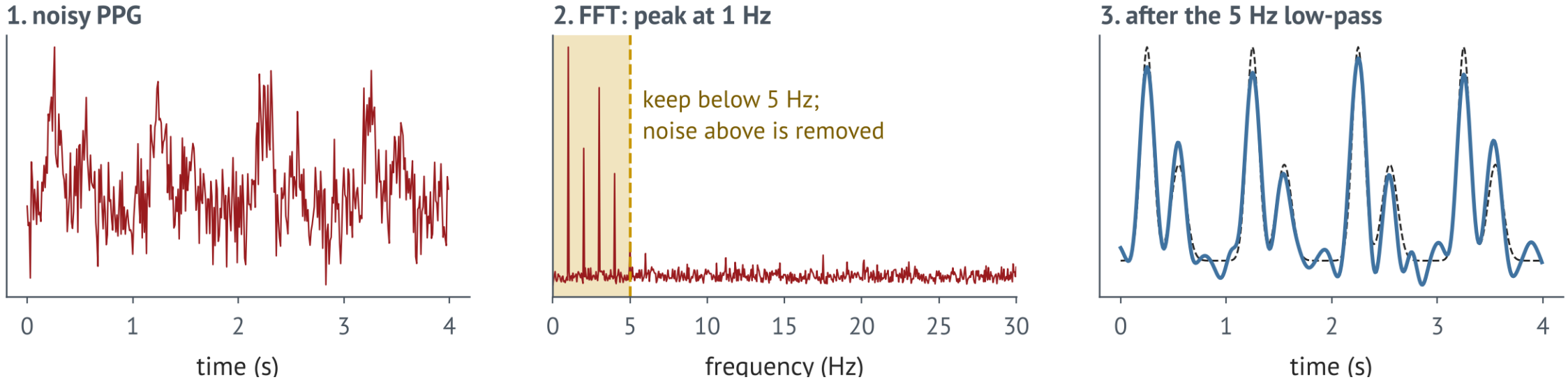
Charge comes in discrete electrons

Grows as √I, not with T

Heat matters: in the class example a 3 µA photodiode signal read 4 µA at 100 °C.

Filtering noise: a PPG example

A heartbeat signal lives near 1 Hz; a low-pass filter at 5 Hz removes the faster noise.



Signal band

$$f_{\text{heart}} \approx 1 \text{ Hz}$$

60 beats per minute, plus a few harmonics

Filter

$$f_{\text{cutoff}} = 5 \text{ Hz}$$

Passes the pulse shape, blocks faster noise

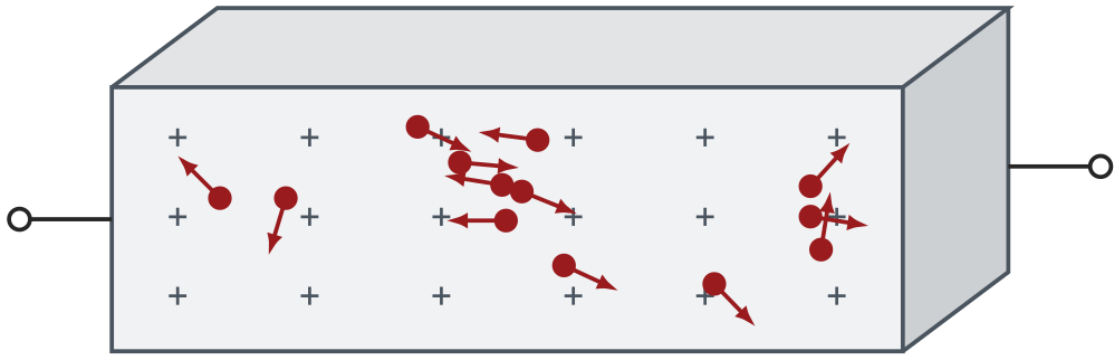
Result

noisy → LPF → clean

Dashed: the true pulse; blue: filtered

Thermal Noise

Carriers in a resistor jiggle with heat, so its voltage fluctuates even when no current flows.



$$v_{n,\text{rms}} = \sqrt{4kTRB}$$

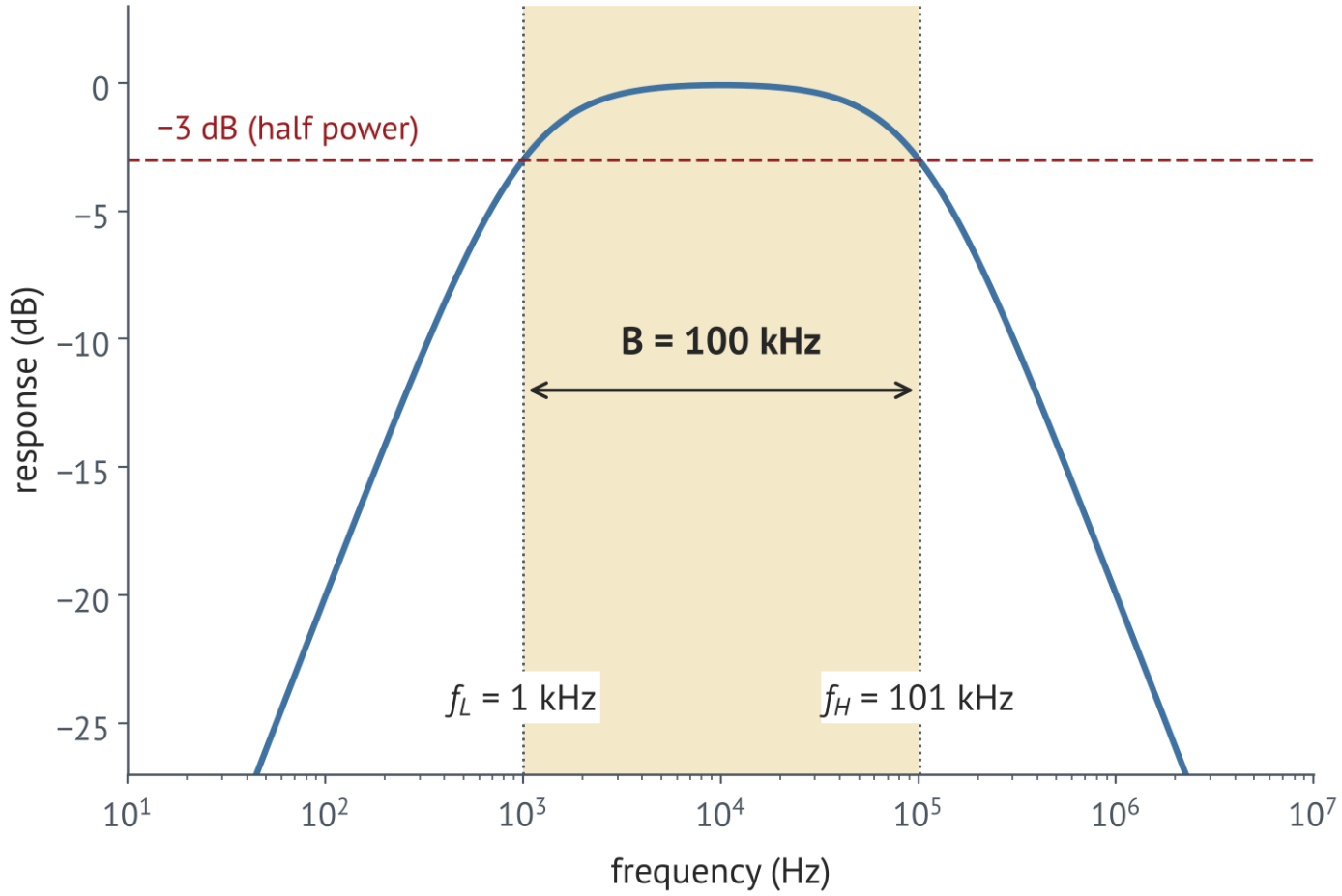
Symbol	Meaning
$v_{n,\text{rms}}$	RMS noise voltage across the resistor (V)
k	Boltzmann constant, 1.38×10^{-23} J/K
T	absolute temperature (K); room ≈ 300 K
R	resistance (Ω): larger R, more noise
B	noise bandwidth (Hz): larger B, more noise

Example: 1 kΩ at 300 K over B = 100 kHz

$v_n = \sqrt{(4 \times 1.38 \times 10^{-23} \times 300 \times 1000 \times 10^5)}$ = **1.29 µV**

Bandwidth

Bandwidth B is the width of the band a system passes, measured between its −3 dB points.



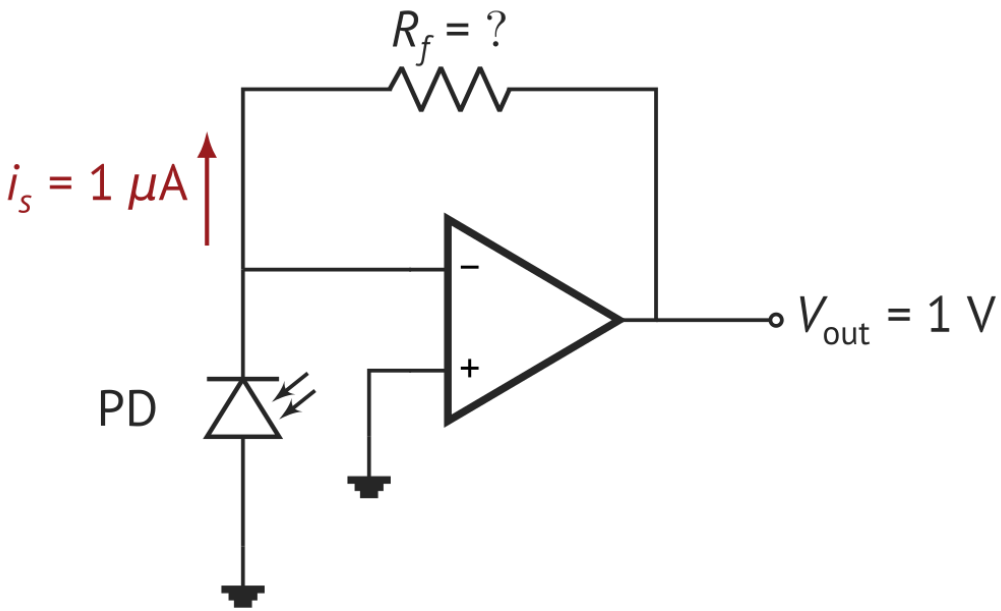
Bandwidth

$$B = f_H - f_L = 101 - 1 = 100 \text{ kHz}$$

- f_L, f_H: where the response is down **3 dB** (half power)
- Noise power is **proportional to B**
- B in √(4kTRB) is this width, in Hz

Thermal Noise: example

A TIA that turns 1 µA into 1 V needs R_f = 1 MΩ, and that resistor adds about 41 µV of noise.



EXAMPLE

i_s = 1 µA, V_{out} = 1 V, T = 300 K, B = 100 kHz. Find R_f, its noise, and the SNR.

1. $R_f = V_{out}/i_s$ = 1 V / 1 µA = **1 MΩ**
2. $v_{th} = \sqrt{(4 \times 1.38 \times 10^{-23} \times 300 \times 10^6 \times 10^5)}$ = **40.7 µV**
3. SNR = $(1\text{ V} / 40.7\ \mu\text{V})^2 = 6.0 \times 10^8$ = **87.8 dB**

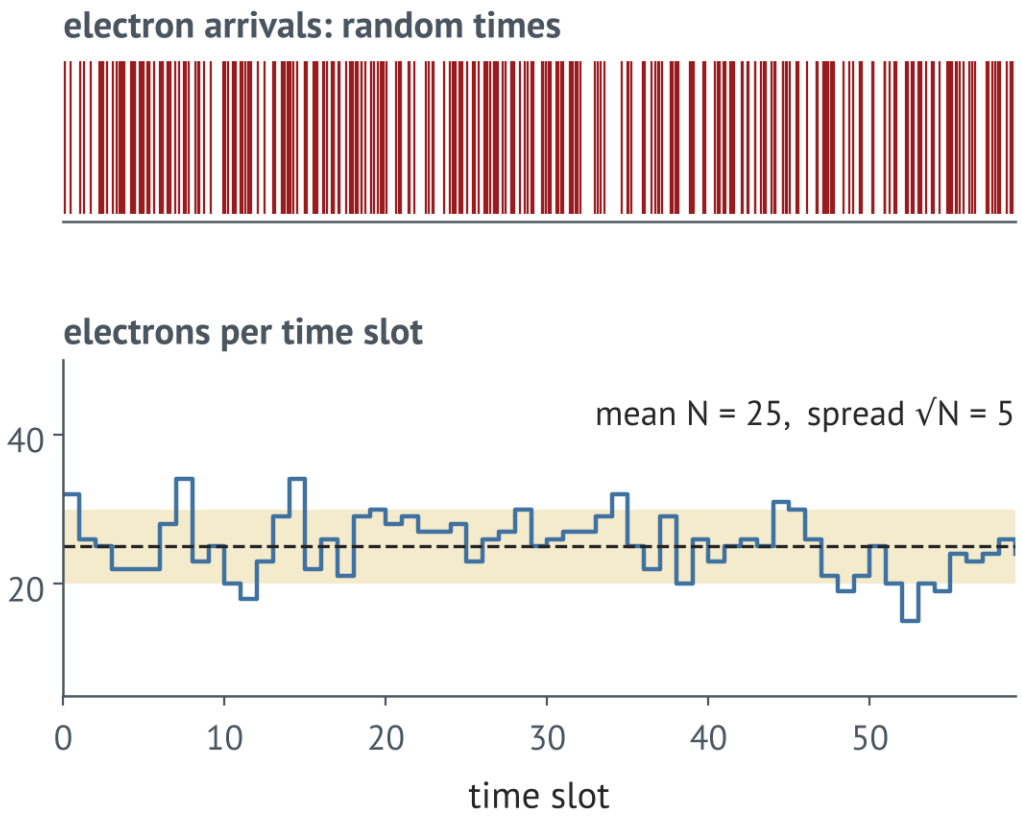
Larger R_f: noise grows as $\sqrt{R_f}$, signal as R_f, so SNR improves.

Thermal noise of the feedback resistor

$$v_{th} = \sqrt{4kTR_fB} = \sqrt{1.656 \times 10^{-9}\ \text{V}^2} = 40.7\ \mu\text{V}$$

Shot Noise

Current is made of discrete charges arriving at random, so even a steady current fluctuates.



Poisson statistics: a count with mean N fluctuates by about √N.

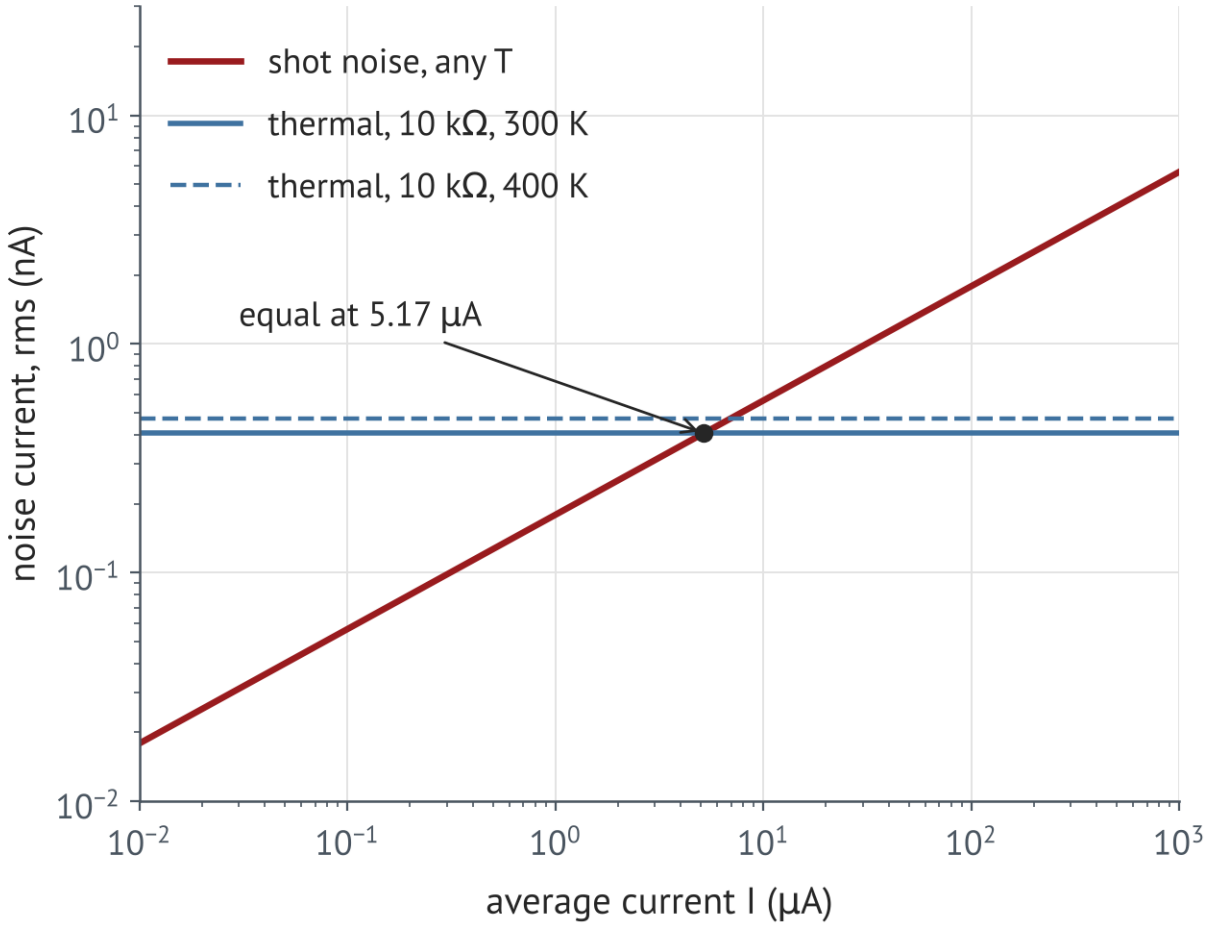
$$i_{\text{shot,rms}} = \sqrt{2\,q\,I\,B}$$

Symbol	Meaning
q	unit charge, 1.602×10^{-19} C
I	average current (A): photocurrent + dark current
B	bandwidth (Hz)

- Does **not** depend on temperature
- Grows as √I and √B
- Seen in PN junctions, vacuum diodes, photodetectors

Shot Noise: key characteristics

Shot noise power grows with the current, so its RMS value grows as √I, at any temperature.



Proportional to current

Noise **power** ∝ I, so the RMS value ∝ √I

Statistical

Poisson arrivals of independent charges

Independent of temperature

Heat changes thermal noise, not shot noise

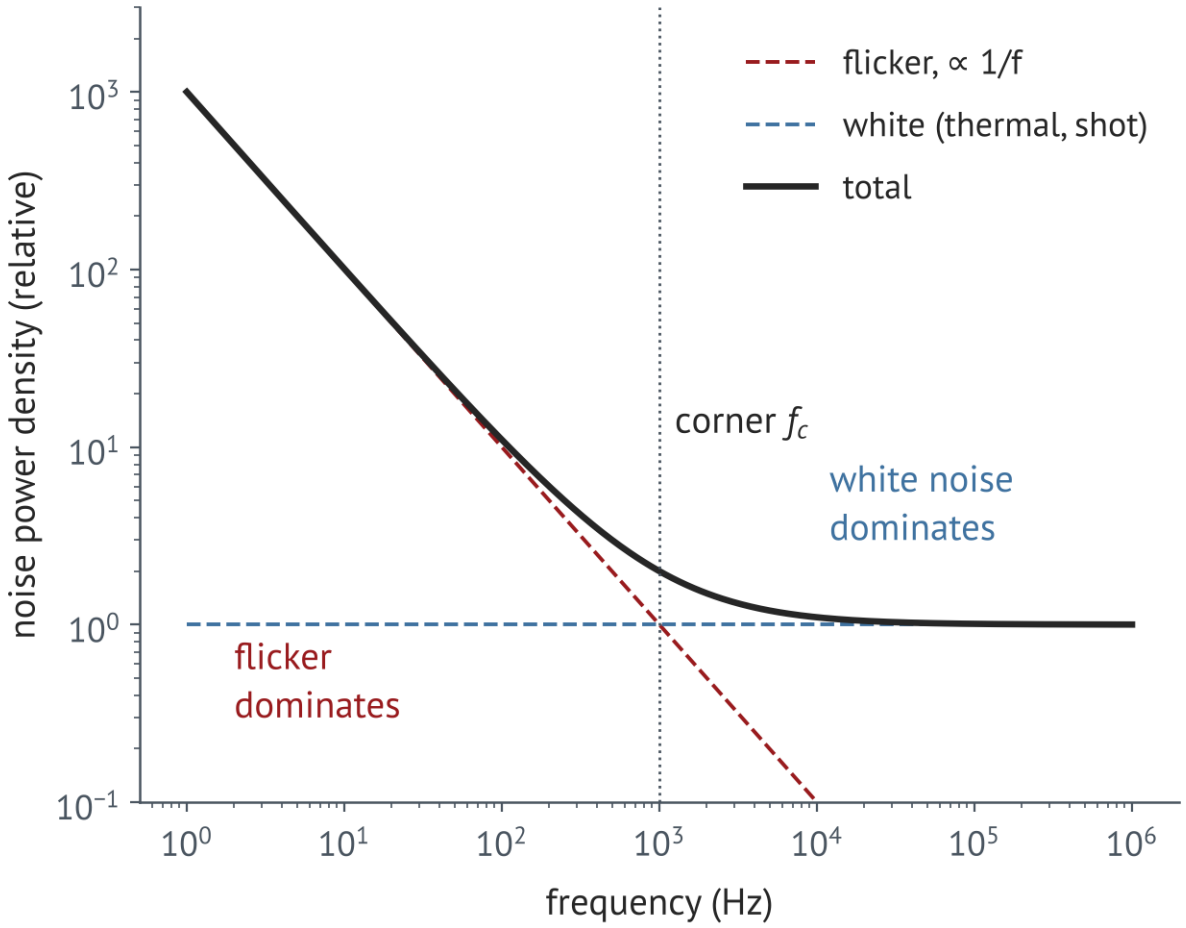
Quantum origin

A direct result of charge coming in units of q

B = 100 kHz. Crossover: I = 2kT/(qR) = 5.17 µA.

Flicker Noise

Flicker (1/f) noise comes from defects and traps; it dominates at low frequency.



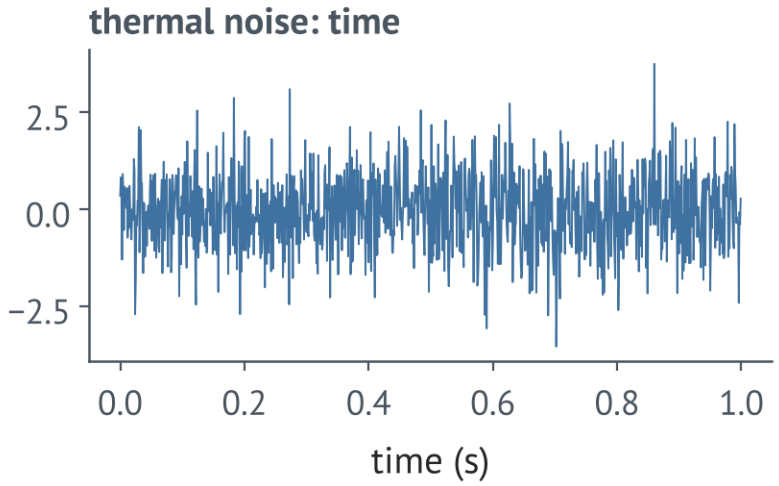
Power spectral density

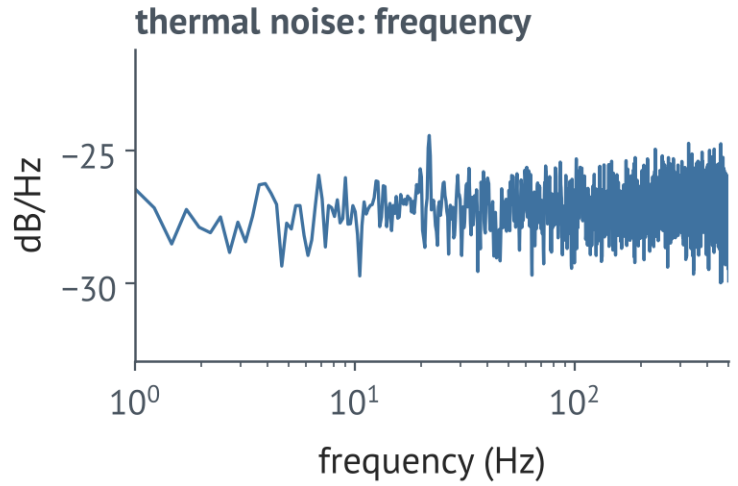
$$S(f) \propto \frac{1}{f^{\alpha}}, \quad \alpha \approx 0.8 \text{ to } 1.2$$

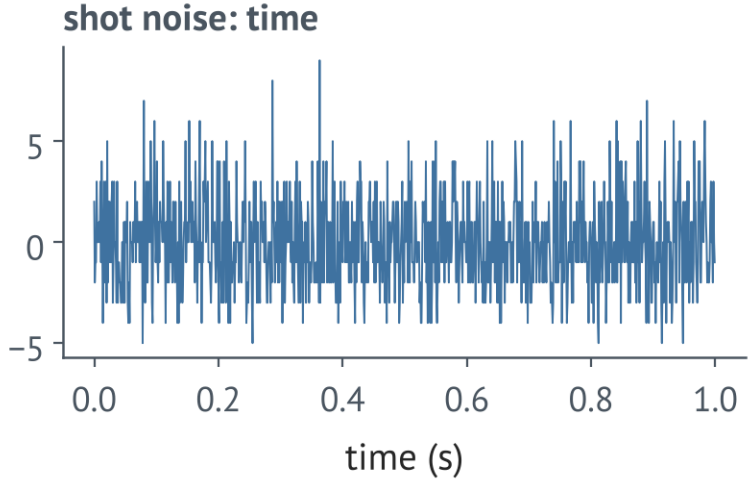
- **Source:** defects, impurities and surface traps in the material
- **Low-frequency dominance:** largest near DC, negligible above the corner
- **Seen in** resistors, transistors, op amps, LEDs
- **Remedy:** move the signal to a higher frequency

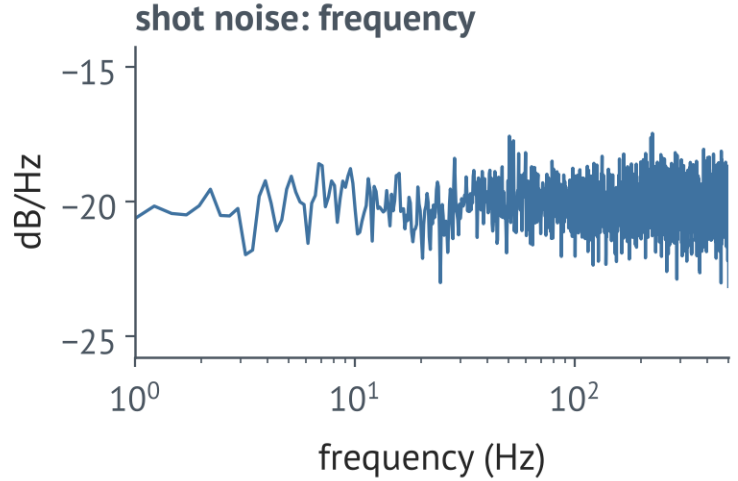
Time and Frequency Domain Visualizations

Thermal and shot noise are white: random in time, with a flat spectrum.





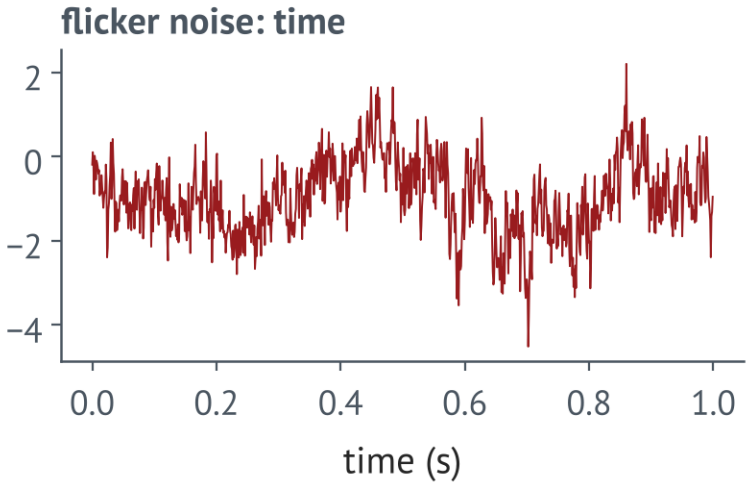


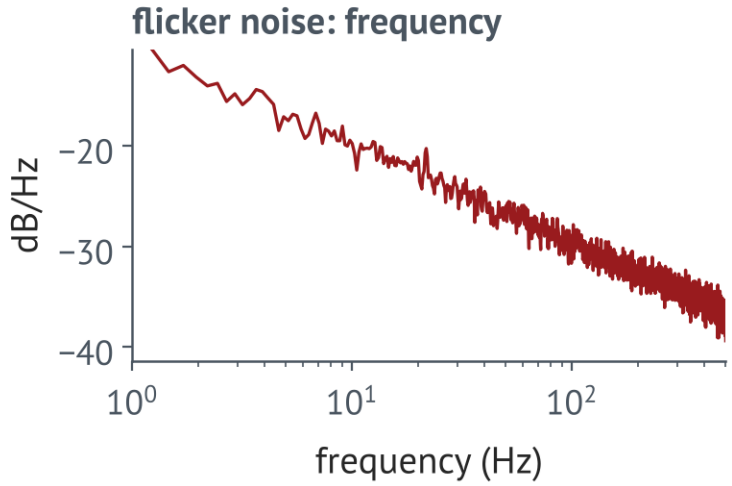


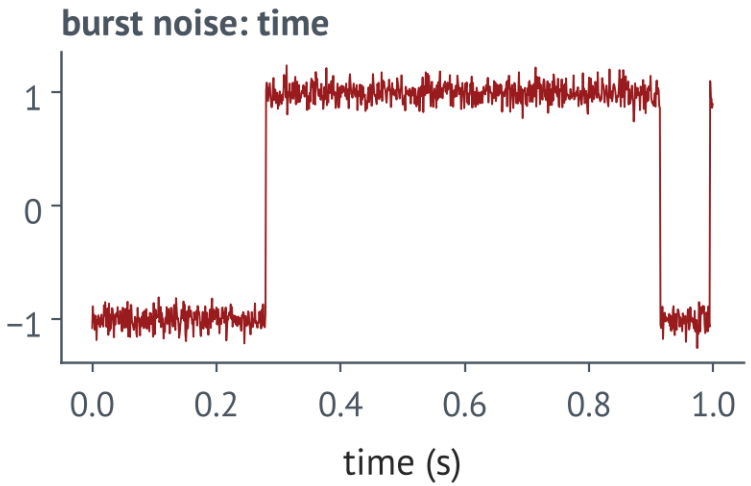
- **Thermal:** Gaussian, centered on 0
- **Shot:** counts of random arrivals around their mean
- Both spectra are **flat** (white): same power per hertz
- So noise power $\propto$ bandwidth B

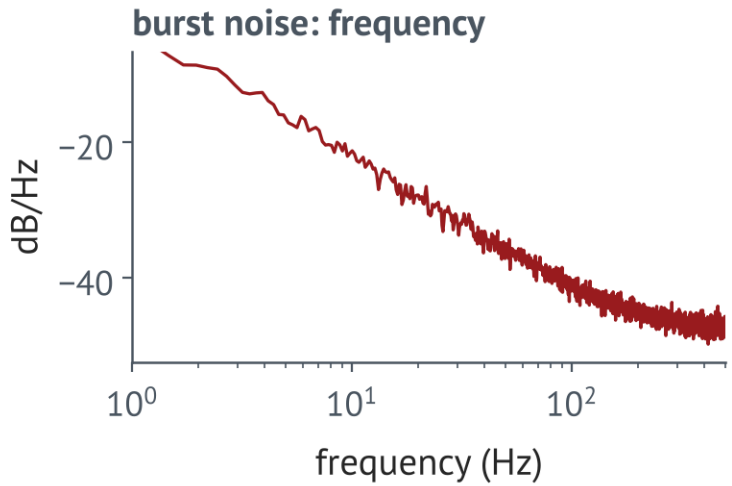
Time and Frequency Domain Visualizations

Flicker noise wanders slowly and piles up at low frequency; burst noise jumps between levels.





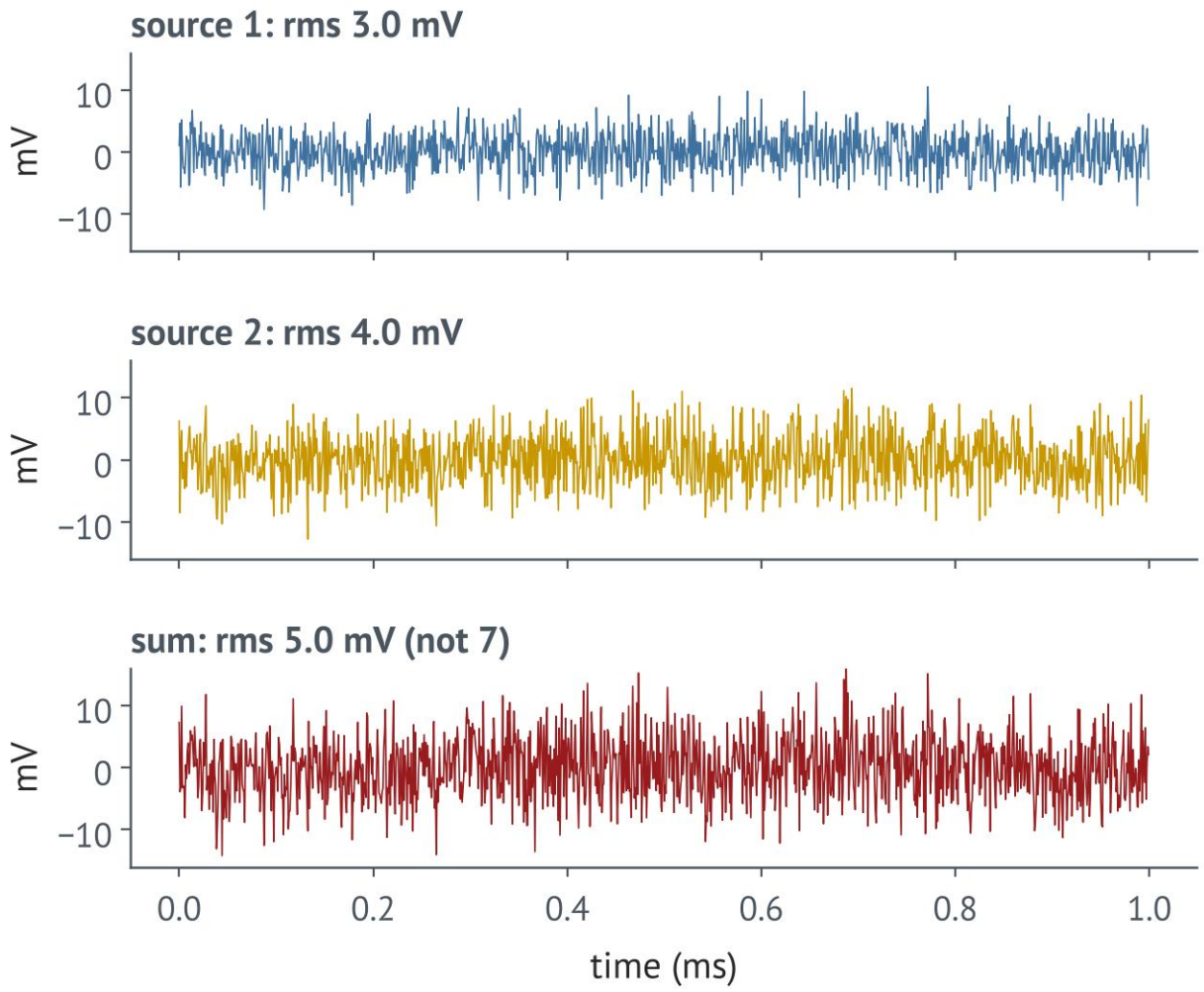




- **Flicker:** slow wander; spectrum falls about 10 dB per decade (1/f)
- **Burst (popcorn):** random jumps between two levels as a trap fills and empties
- Both are worst at **low** frequency

How do different noise sources add?

Independent noise sources add in power, so their RMS values combine as a root-sum-square.



Root-sum-square (RSS)

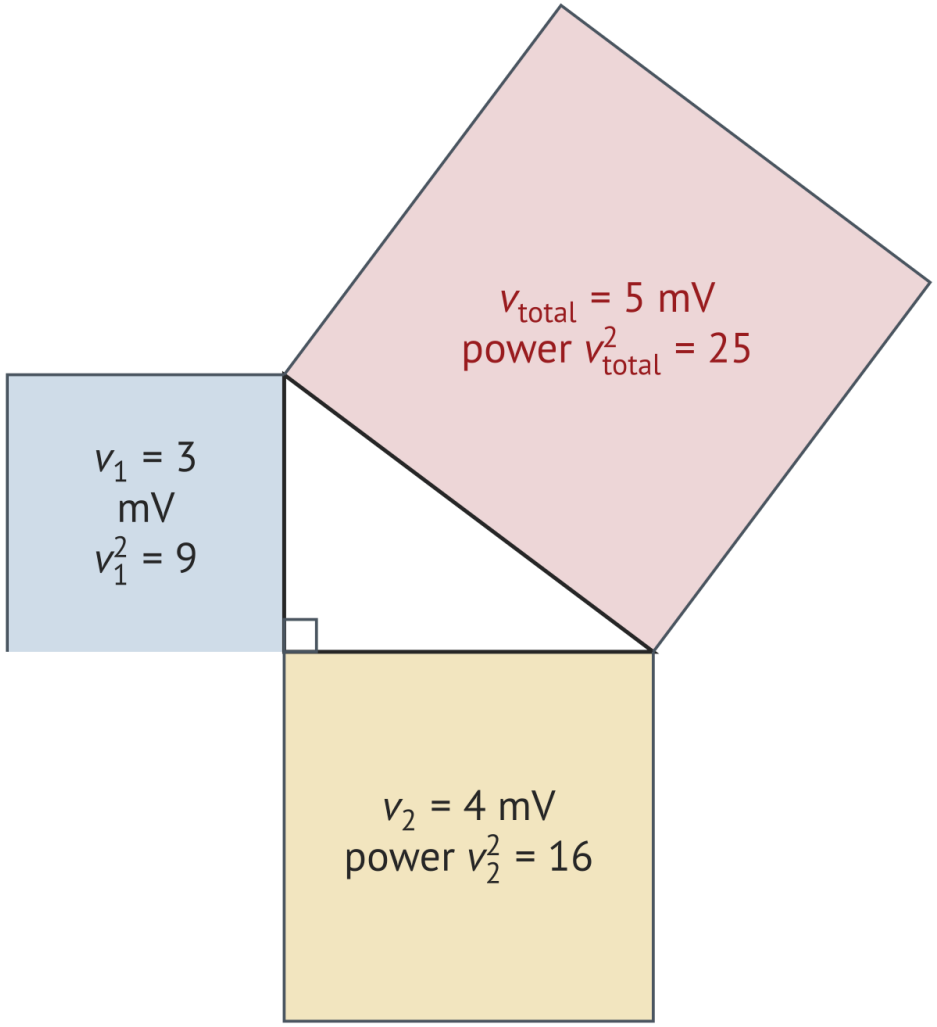
$$V_{n,\text{total}} = \sqrt{V_{n1}^2 + V_{n2}^2 + \cdots + V_{nk}^2}$$

- Uncorrelated sources sometimes add and sometimes cancel
- Their **variances** (powers) add, not their amplitudes
- The largest source dominates

Total: $V_n = \sqrt{(V_{th}^2 + V_{shot}^2 + V_{flicker}^2)}$

Power perspective of noise addition

Noise power goes as the RMS value squared, so adding powers gives the root-sum-square rule.



Power summation (variance) approach

Powers of uncorrelated sources add:

$$P_{total} = P_1 + P_2 + \cdots + P_k$$

Power ∝ (RMS value)², so

$$V_{n,total}^2 = V_{n1}^2 + V_{n2}^2 + \cdots + V_{nk}^2$$

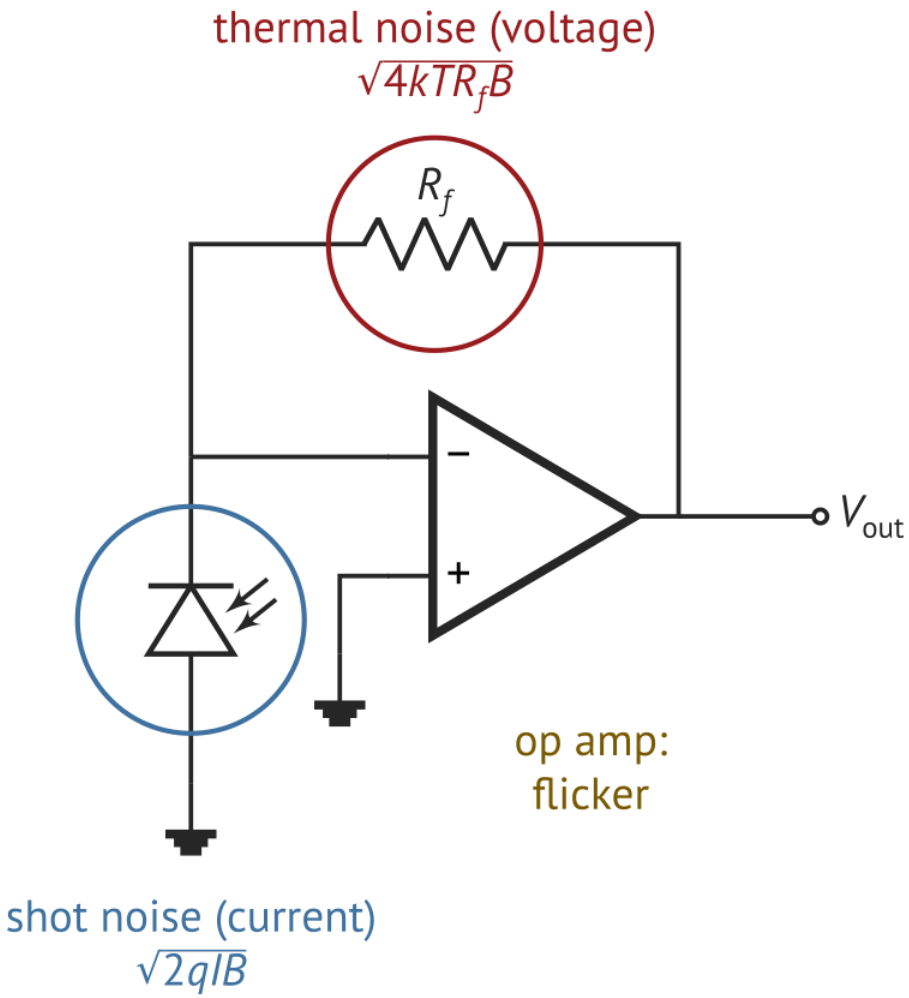
Take the square root:

$$V_{n,total} = \sqrt{V_{n1}^2 + V_{n2}^2 + \cdots + V_{nk}^2}$$

Like Pythagoras: 3 mV and 4 mV of independent noise make 5 mV.

Examples of calculating Noise

In a photodiode receiver, R_f adds thermal noise and the photocurrent adds shot noise.



1. **Thermal** (voltage), mostly from R_f: $v_{th} = \sqrt{(4kTR_fB)}$
2. **Shot** (current), from the photodiode current I: $i_{sh} = \sqrt{(2qIB)}$
3. The TIA turns the shot current into a voltage: $v_{sh} = i_{sh}R_f = R_f\sqrt{(2qIB)}$
4. **Flicker** from the op amp: v_{fl}, if it matters

Total output noise

$$v_n = \sqrt{v_{th}^2 + v_{sh}^2 + v_{fl}^2}$$

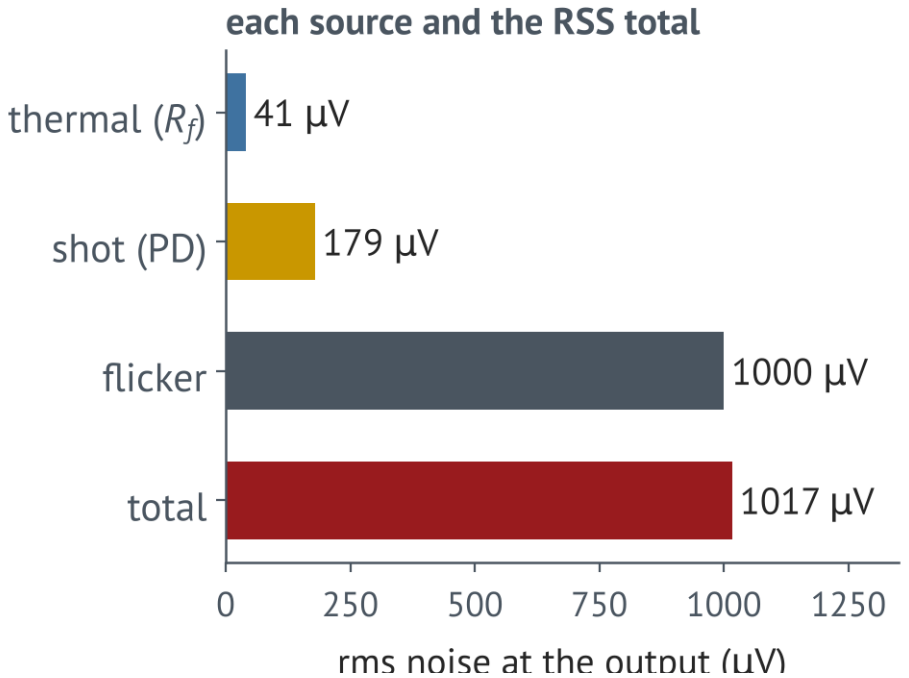
Examples of calculating Noise: numbers

With R_f = 1 MΩ and 1 μA, shot noise beats thermal noise, but 1 mV of flicker swamps both.

EXAMPLE

R_f = 1 MΩ, I = 1 μA, T = 300 K, B = 100 kHz, flicker 1 mV at the output.

1. $v_{th} = \sqrt{(4kTR_fB)} = \sqrt{(1.656 \times 10^{-9})}$ V = **40.7 μV**
2. $i_{sh} = \sqrt{(2qIB)} = \sqrt{(3.20 \times 10^{-20})}$ A = 0.179 nA, so $v_{sh} = i_{sh}R_f$ = **179 μV**
3. Without flicker: $\sqrt{(40.7^2 + 179^2)}$ μV = **184 μV**
4. With flicker: $\sqrt{(0.0407^2 + 0.179^2 + 1^2)}$ mV = **1.02 mV**



$$\text{SNR} = \frac{(1\text{ V})^2}{(1.02\text{ mV})^2} = 9.7 \times 10^5 = 59.9\text{ dB}$$

The total is barely above the largest term: fix the biggest source first.

Power perspective of noise addition: example

5 mV, 3 mV and 2 mV of independent noise add up to 6.16 mV, not 10 mV.

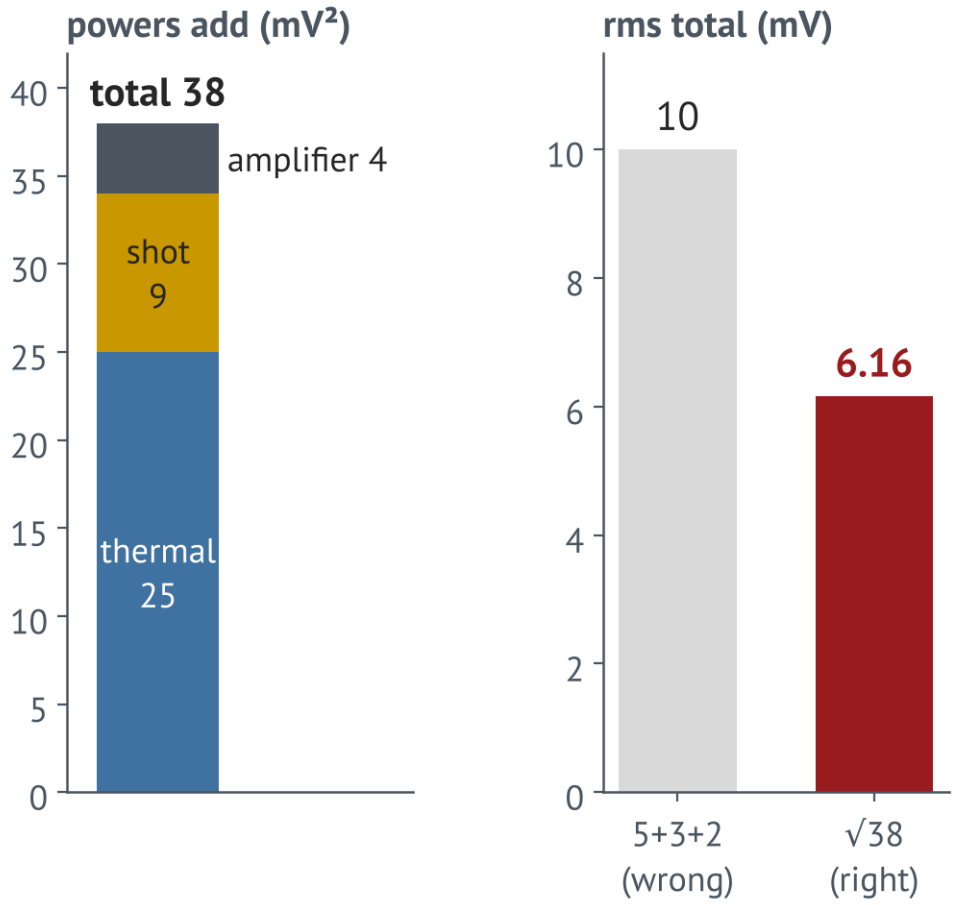
EXAMPLE

A photodetector system has thermal noise from a resistor, **5 mV**; shot noise from the photodiode current, **3 mV**; amplifier noise from the TIA, **2 mV**. Find the total RMS noise.

1. Square each: $25 + 9 + 4 = 38\ \text{mV}^2$
2. Square root: $\sqrt{38}$ mV = **6.16 mV**

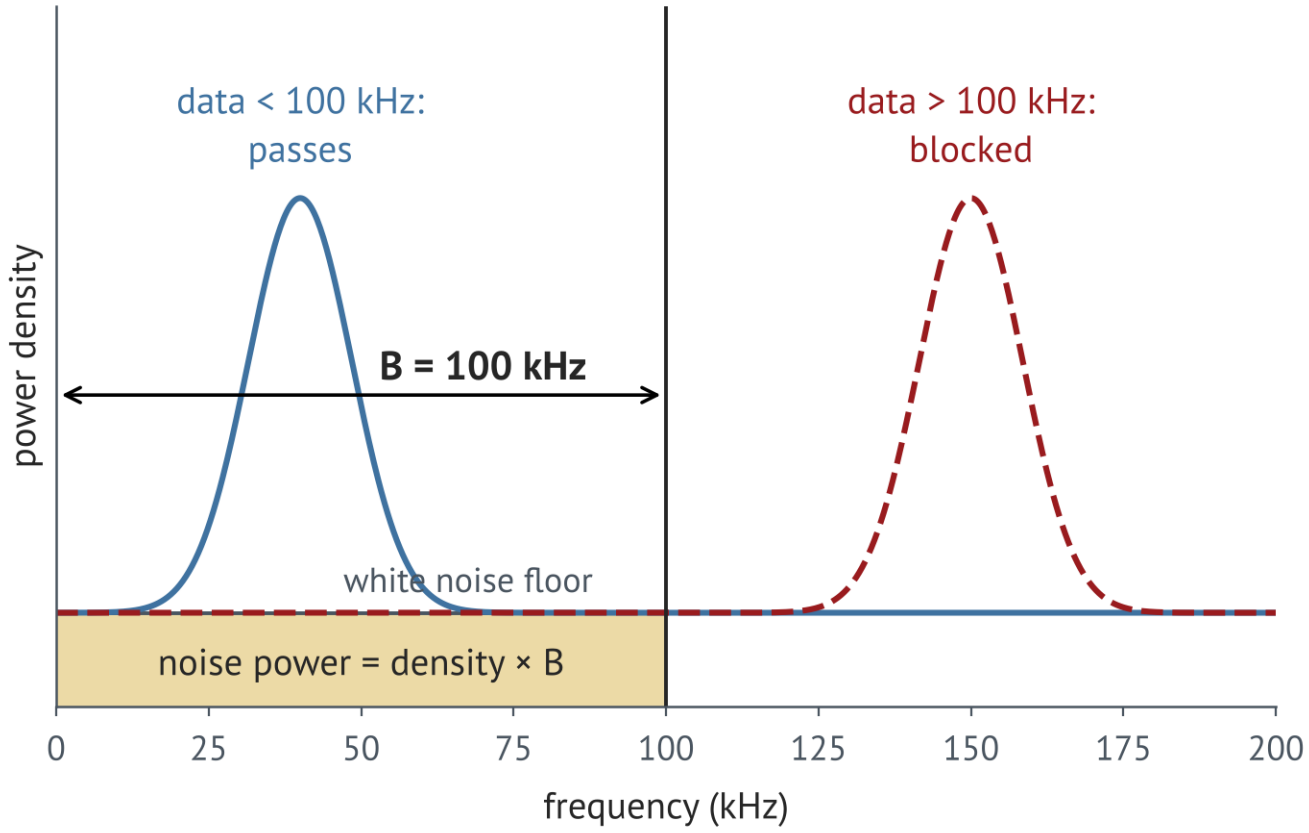
Total RMS noise

$$V_{n,\text{total}} = \sqrt{(5\ \text{mV})^2 + (3\ \text{mV})^2 + (2\ \text{mV})^2}$$
$$= \sqrt{25 + 9 + 4}\ \text{mV} = \sqrt{38}\ \text{mV} \approx 6.16\ \text{mV}$$



How do we calculate B

B is the range of frequencies that reaches the measurement; noise power grows in proportion to it.



A receiver that passes 0 to 100 kHz: the shaded area is the noise it lets in.

Noise power ∝ bandwidth

$$P_n = S_n \times B$$

- **B (or Δf):** the frequency range a system passes
- In noise formulas: the range through which noise reaches you
- Choose B just wide enough for the data

Double B: double the noise power, √2 times the RMS noise.

How do we calculate Δf / B

Use the bandwidth of whatever limits the band: the signal, the amplifier, a filter, or the instrument.



Signal bandwidth

The band the signal occupies

Example: Radio signal over 10 kHz: Δf = 10 kHz

Amplifier bandwidth

An amplifier passes noise up to its −3 dB bandwidth

Example: TIA with 1 MHz: Δf = 1 MHz

Filter bandwidth

A filter cuts noise outside its band

Example: Low-pass at 100 kHz: Δf = 100 kHz

Instrument bandwidth

A scope or analyzer sees only its own band

Example: 20 MHz oscilloscope: Δf = 20 MHz

Sampled data: samples hold frequencies only up to $f_s/2$ (Nyquist), so sampling at 200 kHz gives B = 100 kHz.

Example

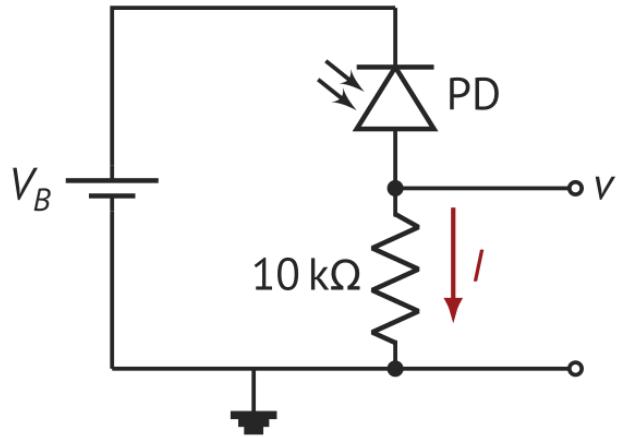
A reverse-biased photodiode drives a 10 kΩ resistor: find the noise sources and the RMS noise.

EXAMPLE

A photodetector in reverse bias, with light shining on it, is connected to a **10 kΩ** resistor. We measure the voltage across the resistor.

A) What are the sources of noise in this voltage?

B) With **1 mA** of photocurrent, what is the RMS noise voltage across the resistor? The voltage is sampled at **200 kHz** (a bandwidth of 100 kHz).



Given

- Photocurrent I = 1 mA = 10^{-3} A
- Resistor R = 10 kΩ = 10^4 Ω
- Sampling f_s = 200 kHz, so B = $f_s/2$ = 10^5 Hz
- Room temperature T = 300 K
- Electron charge q = 1.602 × 10^{-19} C
- Boltzmann k = 1.38 × 10^{-23} J/K

Sampling at f_s keeps frequencies up to $f_s/2$ (Nyquist): 200 kHz gives B = 100 kHz.

Example: solution

Shot noise of the 1 mA photocurrent gives 56.6 µV; the resistor adds only 4.07 µV; total ≈ 57 µV.

SOLUTION

A) Shot noise of the photocurrent through R, and thermal noise of R itself (flicker only at low frequency)

1. Shot current: $i_{sh} = \sqrt{(2 \times 1.602 \times 10^{-19} \times 10^{-3} \times 10^{5})} = \sqrt{(3.20 \times 10^{-17})}$ A = **5.66 nA**
2. Shot voltage: $v_{sh} = i_{sh}R$ = 5.66 nA × 10 kΩ = **56.6 µV**
3. Thermal: $v_{th} = \sqrt{(4 \times 1.38 \times 10^{-23} \times 300 \times 10^{4} \times 10^{5})} = \sqrt{(1.656 \times 10^{-11})}$ V = **4.07 µV**
4. Total: $\sqrt{(56.6^2 + 4.07^2)}$ µV = **56.7 µV ≈ 57 µV**

Signal: 1 mA × 10 kΩ = 10 V, so SNR = $(10 \text{ V} / 56.7 \text{ µV})^2 \approx 3 \times 10^{10}$ (105 dB)

Shot noise (current, then voltage)

$$v_{sh} = R\sqrt{2qIB} = 56.6\ \mu V$$

Thermal noise of R

$$v_{th} = \sqrt{4kTRB} = 4.07\ \mu V$$

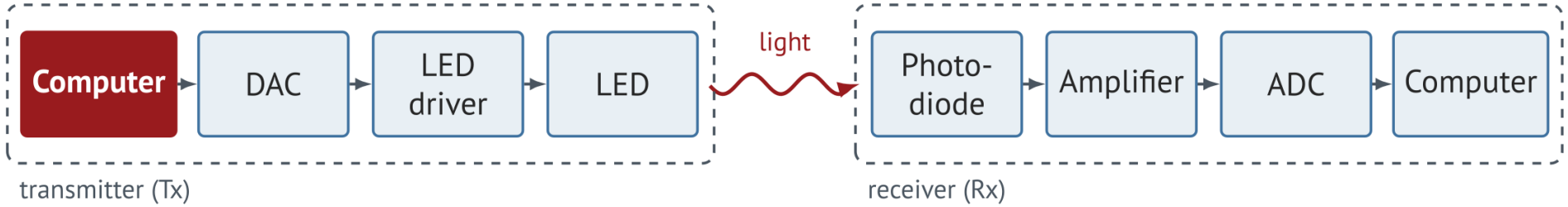
Total (uncorrelated: powers add)

$$v_n = \sqrt{v_{sh}^2 + v_{th}^2} \approx 57\ \mu V$$

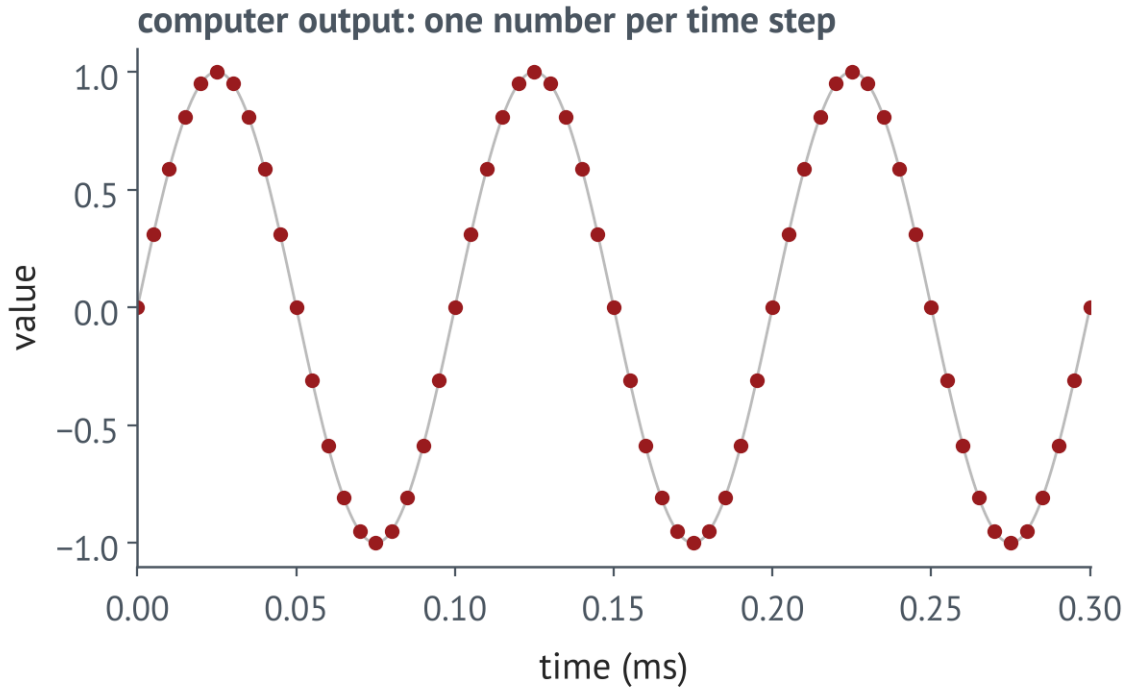
Shot noise dominates at 1 mA

Real world system: the computer

Inside the computer a signal is just an array of numbers, one per time step.



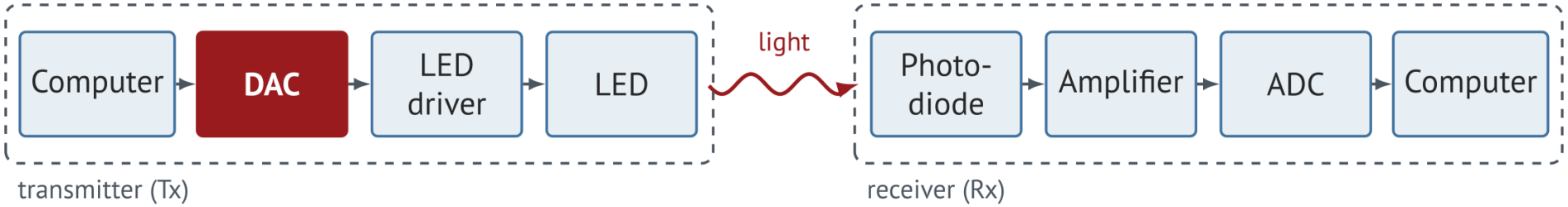
- A sine wave is an array of double-precision numbers, one per time step
- Doubles hold every integer from -2^{53} to 2^{53} exactly: 2^{54} levels
- Still one value per step: quantized in **voltage** and in **time**

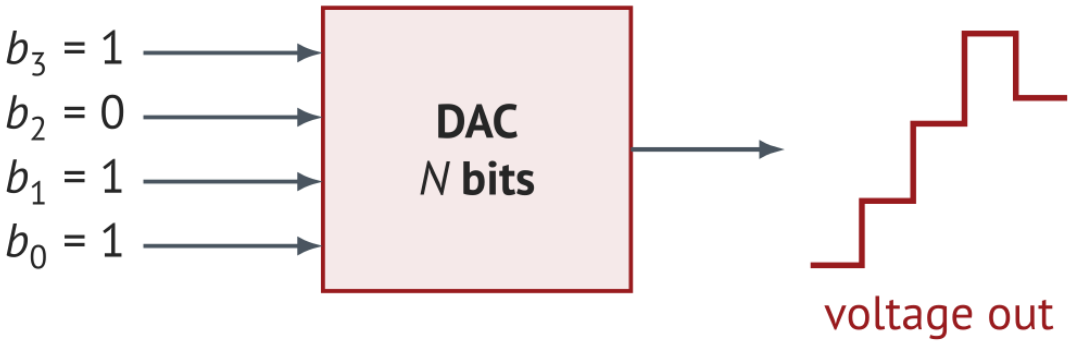


2^{53} = 9,007,199,254,740,992: far finer than any DAC

Real world system: the DAC

A digital-to-analog converter turns each number into a voltage, and adds its own errors.





Output of an N-bit DAC

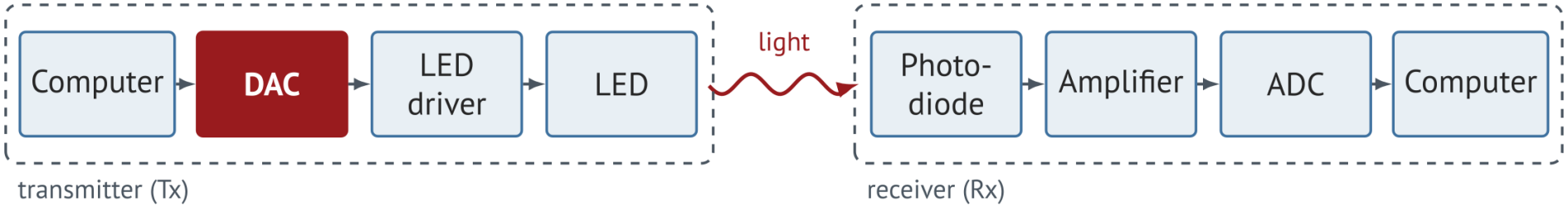
$$V_{out} = \text{code} \times \frac{V_{FS}}{2^N}$$

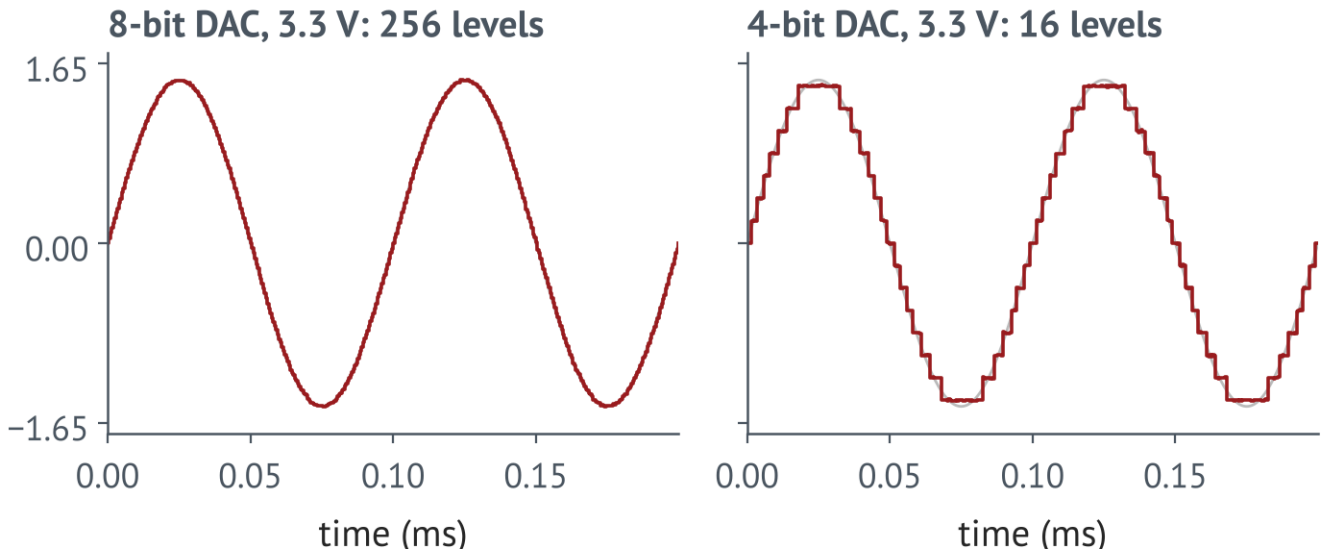
Noise and errors in a DAC

- **Quantization:** the output moves in steps; finer with more bits
- **Thermal:** from its resistors; grows with T and R
- **Nonlinearity:** uneven steps distort the signal (harmonics)
- **Clock jitter:** updates at slightly wrong times; worst at high frequency
- **Power supply:** supply ripple leaks into the output

Real world system: DAC resolution

An N-bit DAC has 2^N levels: on 3.3 V, 8 bits give 12.9 mV steps and 4 bits give 206 mV steps.





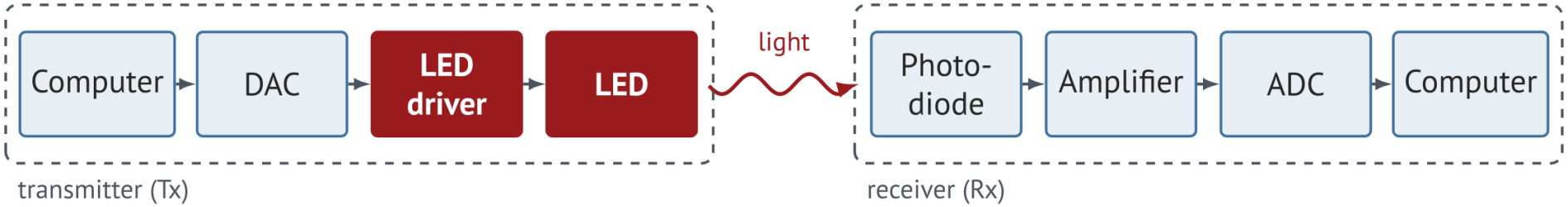
Gray: the ideal sine. The 4-bit staircase is easy to see.

Bits N	Levels 2^N	Step Δ	Noise Δ/√12	Ideal SNR
2	4	825 mV	238 mV	13.8 dB
4	16	206 mV	59.5 mV	25.8 dB
8	256	12.9 mV	3.72 mV	49.9 dB
10	1024	3.22 mV	0.93 mV	62.0 dB

$$\Delta = \frac{V_{FS}}{2^N}, \quad v_q = \frac{\Delta}{\sqrt{12}}, \quad \text{SNR} \approx 6.02N + 1.76 \text{ dB}$$

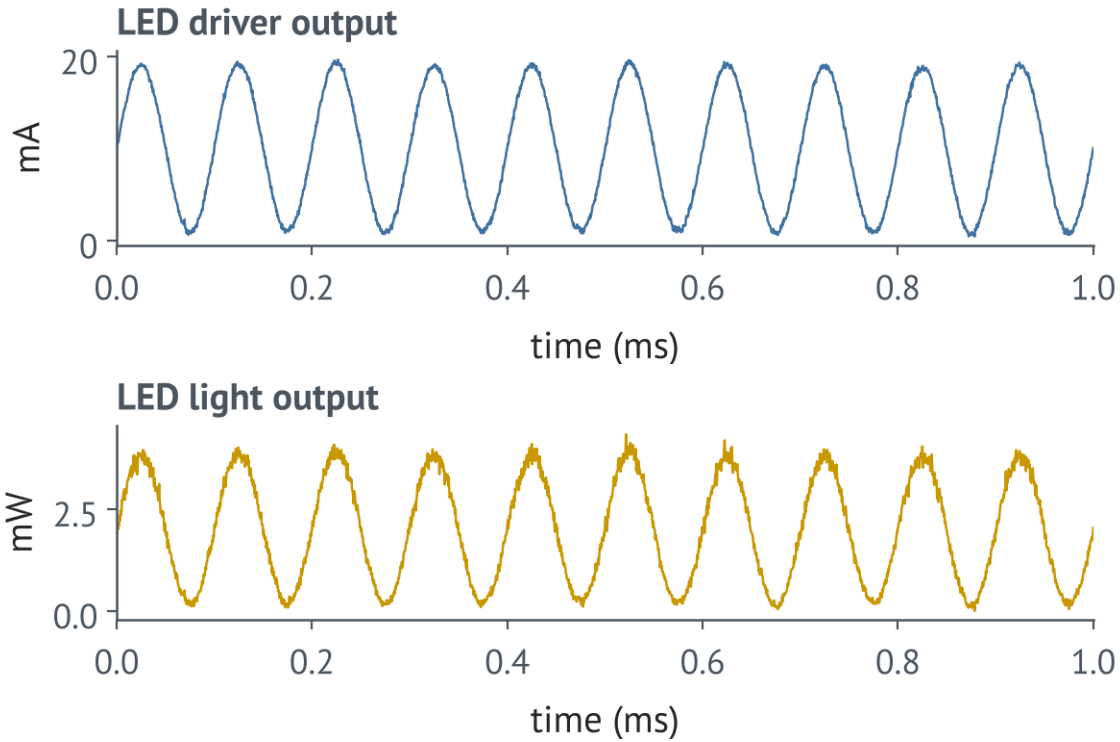
Real world system: LED driver and LED

The driver turns voltage into LED current and the LED turns current into light; both add noise.



- **Thermal:** in the driver's resistors; grows with T and R
- **Shot:** in the LED current; RMS grows as √I
- **Flicker:** in the driver parts; strongest at low frequency

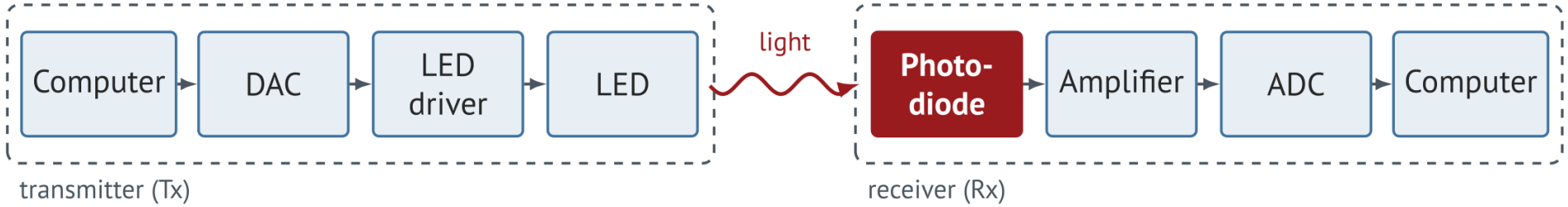
More LED current: the signal grows as I, its shot noise only as √I.



Simulated 10 kHz sine; noise exaggerated to make it visible.

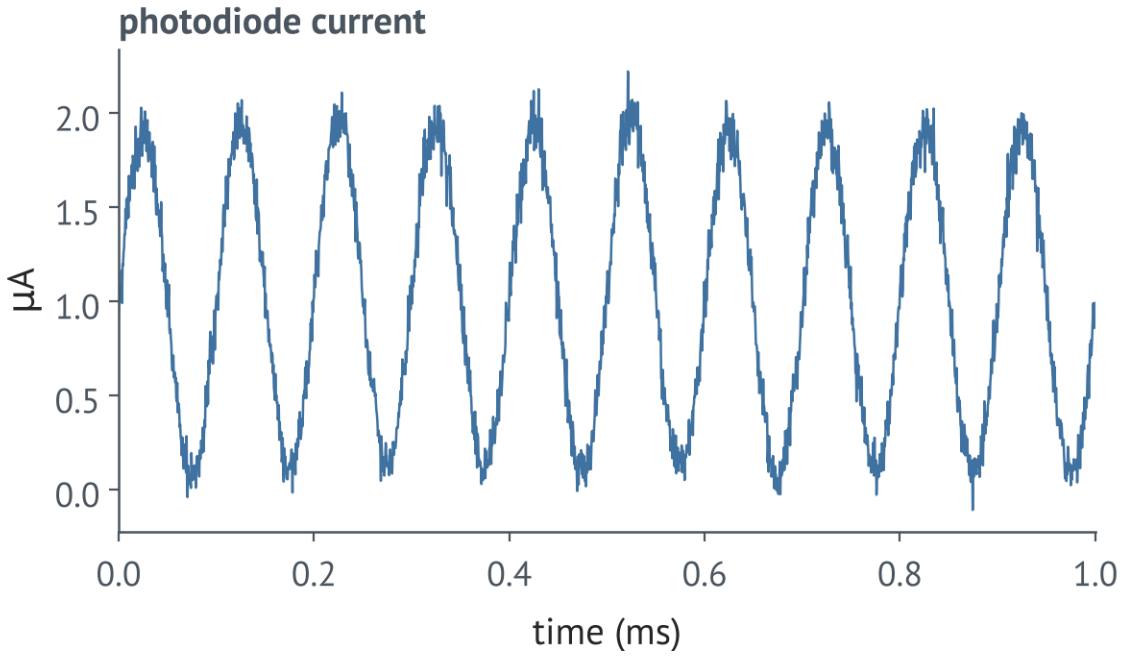
Real world system: the photodetector

The photodiode adds shot noise from photon arrivals, dark current, thermal and flicker noise.



- **Shot:** random photon arrivals; white; matters most at low light
- **Thermal:** resistive parts; present even in the dark
- **Dark current:** heat-generated carriers; grows with T
- **Flicker (1/f):** defects and traps; low frequency

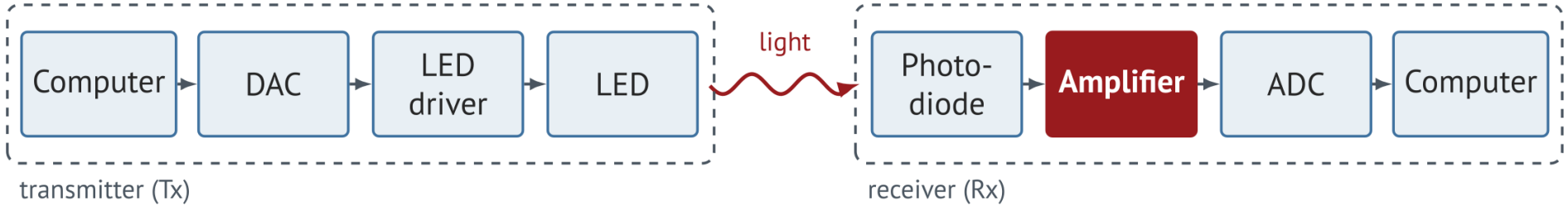
Shot noise relative to the signal falls as 1/√I: more light, cleaner signal.



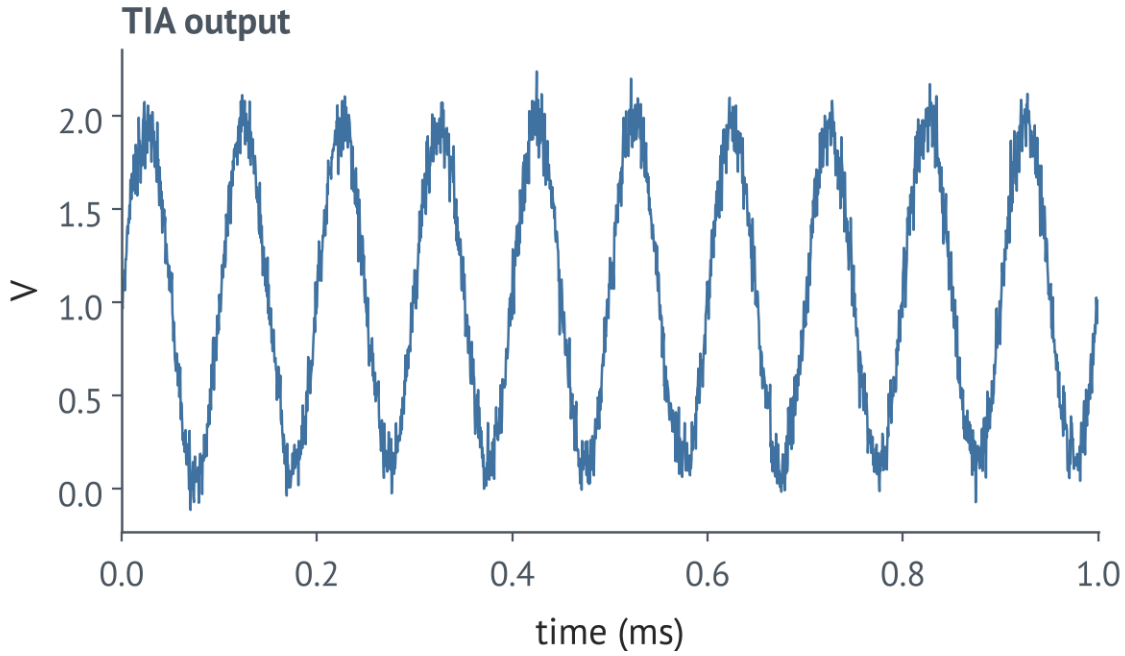
Simulation; noise exaggerated.

Real world system: the amplifier

The TIA adds thermal noise from R_f and op-amp noise, and amplifies the photodiode's shot noise.



- **Thermal, feedback resistor:** appears directly at the output
- **Shot from the photocurrent:** multiplied by R_f
- **Op-amp voltage noise:** amplified by the gain
- **Op-amp current noise:** times the circuit impedance
- **Flicker:** op-amp defects; low frequency

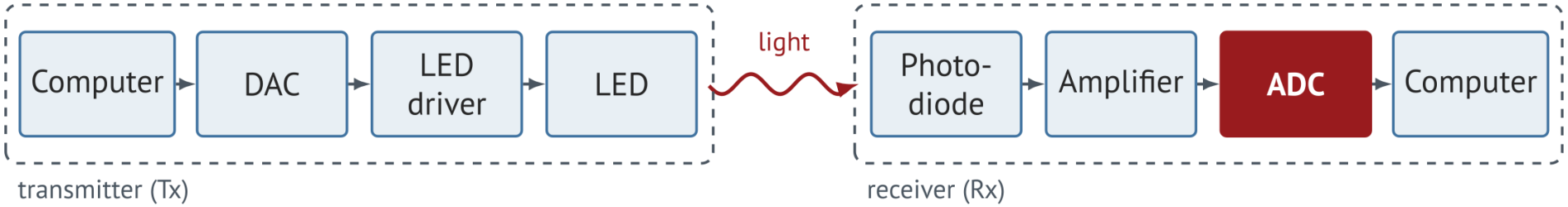


Simulation; noise exaggerated.

The first amplifier after the photodiode must be the quietest: later stages see an already large signal.

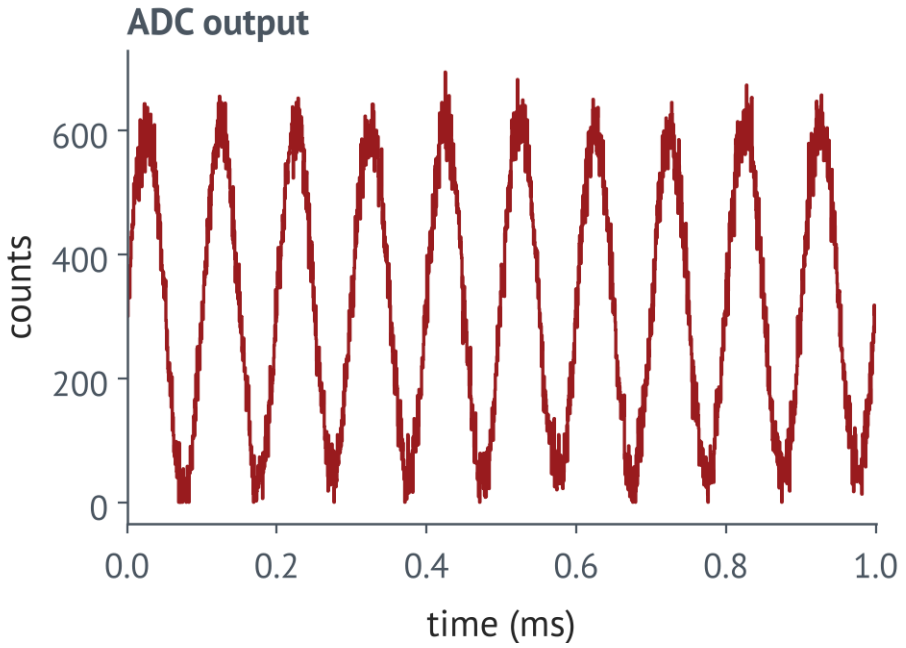
Real world system: the ADC

The ADC rounds each sample to one of 2^N levels and adds timing and reference errors.



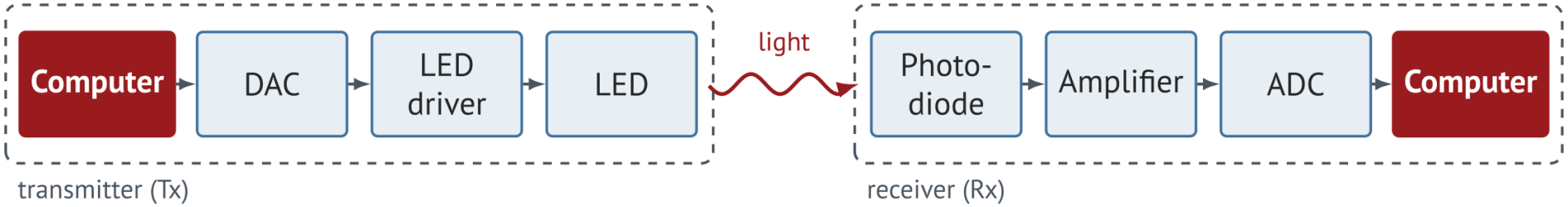
Noise	Cause	Effect
Quantization	rounding to 2^N levels	$\Delta/\sqrt{12}$; halves per extra bit
Thermal	front-end resistors	sets a noise floor
Aperture jitter	sampling-time uncertainty	worst for fast, large signals
Reference	noisy reference voltage	scales every reading
Clock	unstable sampling clock	samples at wrong times
Nonlinearity	uneven transfer curve	harmonic distortion
Flicker (1/f)	semiconductor defects	low-frequency drift

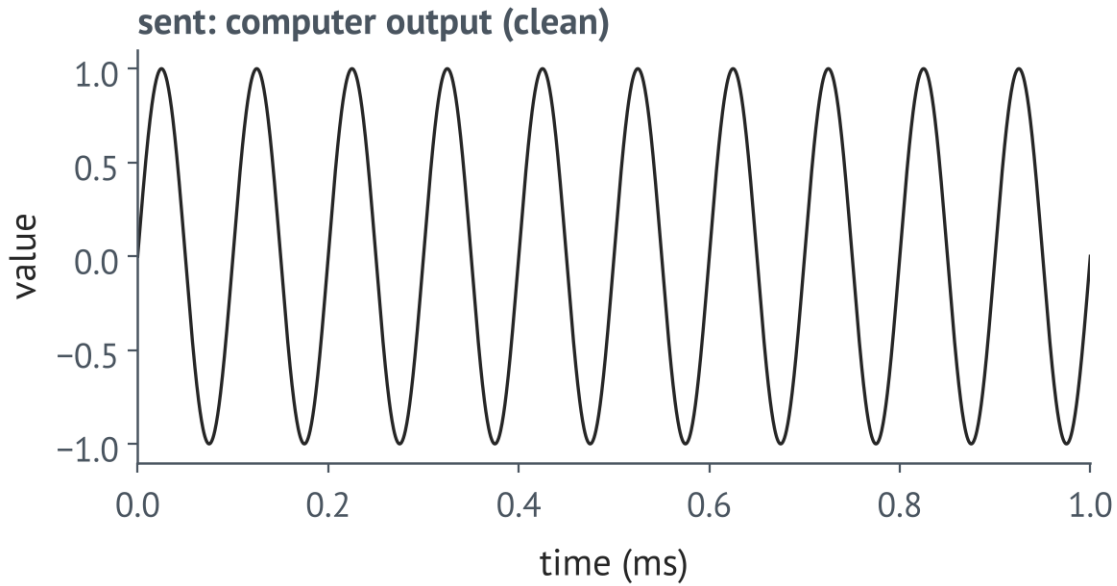
10-bit ADC on 3.3 V: step 3.22 mV, quantization noise 0.93 mV rms

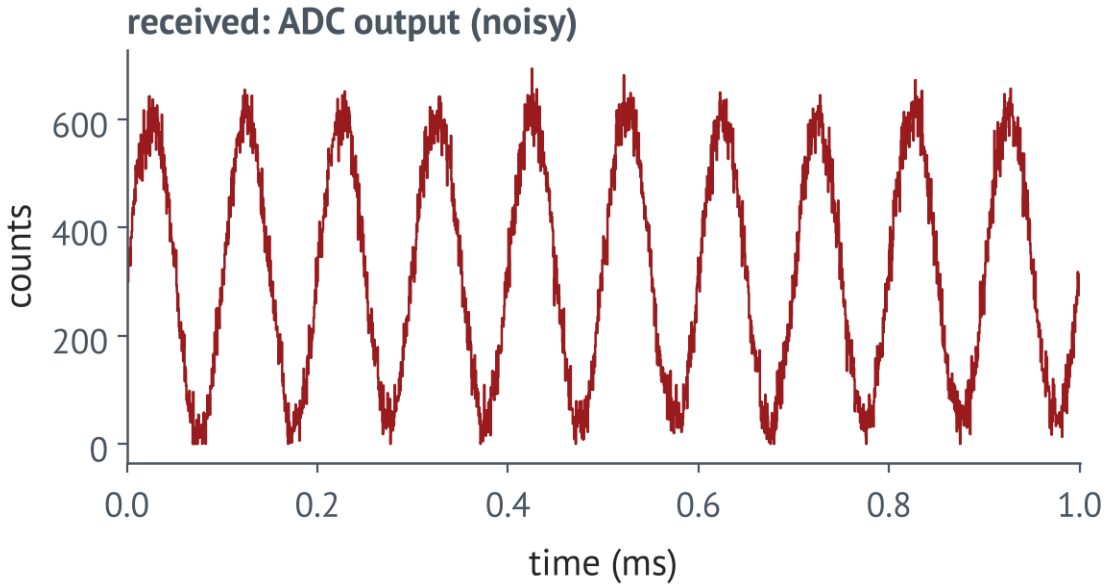


Real world system: end to end

A clean array of numbers goes in; a noisy one comes out, because every block adds a little noise.







To fight it: more signal, a narrower bandwidth, averaging, and coding.