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ECE 105: Introduction to Electrical Engineering

Guest Lecture
Quantum Optics: The Hong-Ou-Mandel (HOM) dip
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A brief introduction to quantum mechanics

"If a tree falls in a forest and no one is around to hear it, does it make a sound?"

- In quantum mechanics, until the tree is ***observed*** and a ***measurement*** is made, it is in a **linear superposition** of two possible **states**:

- Quantum mechanics is distinguished from classical mechanics by the linear algebra of *non-commuting* **observables**.
- *Unlike* classical mechanics, physical properties of systems (e.g., position, momentum) generally ***cannot*** be **measured** to arbitrary accuracy.
- Quantum mechanical systems exhibit key features that distinguish them from everyday classical objects, including:
  - **Wave-particle duality:** local nature of system depends on the observable (how you measure).
  - **Linearity:** system can exist as linear combinations, or superpositions, of states (how you prepare it).
  - **Indistinguishability:** the existence of identical, indistinguishable particles.

A brief introduction to quantum mechanics

Postulate 1: State Vector (Quantum State)

- Any (isolated) system can be fully described by a (column) **state vector** $|\psi\rangle$, which resides in a complex vector space $\mathcal{H}$ called the **Hilbert space**:

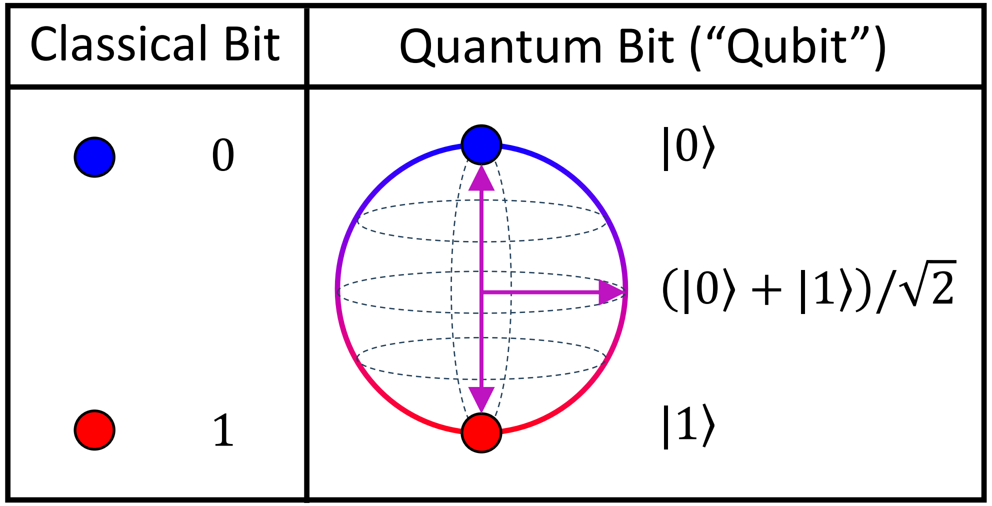
"ket" $$|\psi\rangle = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$ Finite N-dimensional Hilbert space

- The ***Hermitian conjugate*** of a state $|\psi\rangle$ described by a complex *column* vector is given by the complex conjugate transpose, resulting in a complex *row* vector:

"bra" $$\langle\psi| = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}^\dagger = \left(\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}^*\right)^\intercal = \begin{bmatrix} a_1^* & a_2^* & \cdots & a_N^* \end{bmatrix}$$

- *Normalized* complex amplitudes a_n means the "bra-ket" inner product is always equal to **1**:

$$\langle\psi|\psi\rangle = \begin{bmatrix} a_1^* & a_2^* & \cdots & a_N^* \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = |a_1|^2 + |a_2|^2 + \cdots + |a_N|^2 = 1$$



Composite system postulate

- Two *independent* (separable) quantum systems may be described by the **tensor product** of each quantum state:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = |\psi_1\rangle|\psi_2\rangle = |\psi_1, \psi_2\rangle$$

- Example: $|\psi_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|\psi_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 0\begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

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Postulate 2: Observables and Operators

- Every measurable quantity ("observable") in a quantum system is represented by a Hermitian **operator** $\hat{O} = \hat{O}^\dagger$ which acts on the state vector (in the state space $\mathcal{H}$).
- Mathematically, operators are **matrices** in finite-dimensional spaces whose **eigenvectors** form a complete basis and which act on state **vectors**.

Postulate 3: Individual Measurements

- Measuring an observable is equivalent to finding the **eigenvalues** of the corresponding operator.
- Hermitian operators have **real** eigenvalues, which correspond to the *possible* measurement outcomes.
- Consider a finite particle number *basis* $|n\rangle = \{|0\rangle, |1\rangle, |2\rangle, \cdots, |N\rangle\}$, where:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \cdots, |N\rangle = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

- The particle *number* operator $\widehat{N}$ is given by the matrix:

$$\widehat{N} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & N \end{bmatrix}$$

Observable	Classical Symbol	Quantum Operator	Operation
Position	$\mathbf{r}$	$\hat{r}$	Multiply by $\mathbf{r}$
Momentum	$\mathbf{p}$	$\hat{p}$	$-i\hbar\nabla = -i\hbar\left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)$
Kinetic energy	T	$\hat{T}$	$-\frac{\hbar^2\nabla^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

- Number states are **eigenstates** of the (Hermitian) number operator, in which the (real) eigenvalues are the particle numbers: $\widehat{N}|n\rangle = n|n\rangle$
- **Example**: $\widehat{N}|2\rangle = 2|2\rangle$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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Postulate 2: Observables and operators, continued...

- Creation operator ($\hat{b}^\dagger$): Increases particle number in state by 1

Matrix elements are *transition probability* ***amplitudes***

$$\hat{b}^\dagger = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{1} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix}$$

$$\begin{array}{c|cccc} & |0\rangle & |1\rangle & |2\rangle & |3\rangle \\ \hline \langle 0| & 0 & 0 & 0 & 0 \\ \langle 1| & \sqrt{1} & 0 & 0 & 0 \\ \langle 2| & 0 & \sqrt{2} & 0 & 0 \\ \langle 3| & 0 & 0 & \sqrt{3} & 0 \end{array}$$

$$\hat{b}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

- Annihilation operator ($\hat{b}$): Decreases particle number in state by 1

$$\hat{b} = \begin{bmatrix} 0 & \sqrt{1} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c|cccc} & |0\rangle & |1\rangle & |2\rangle & |3\rangle \\ \hline \langle 0| & 0 & \sqrt{1} & 0 & 0 \\ \langle 1| & 0 & 0 & \sqrt{2} & 0 \\ \langle 2| & 0 & 0 & 0 & \sqrt{3} \\ \langle 3| & 0 & 0 & 0 & 0 \end{array}$$

$$\hat{b}|n\rangle = \sqrt{n}|n-1\rangle$$

- Number operator ($\hat{N}$): Yields particle number in state

$$\hat{N} = \hat{b}^\dagger\hat{b} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{1} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix}\begin{bmatrix} 0 & \sqrt{1} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\hat{N}|n\rangle = \hat{b}^\dagger\big(\hat{b}|n\rangle\big) = \hat{b}^\dagger(\sqrt{n}|n-1\rangle)$$
$$= \sqrt{n}\big(\hat{b}^\dagger|n-1\rangle\big) = \sqrt{n}(\sqrt{n}|n\rangle) = n|n\rangle$$

The HOM effect: Interference between 2 indistinguishable photons

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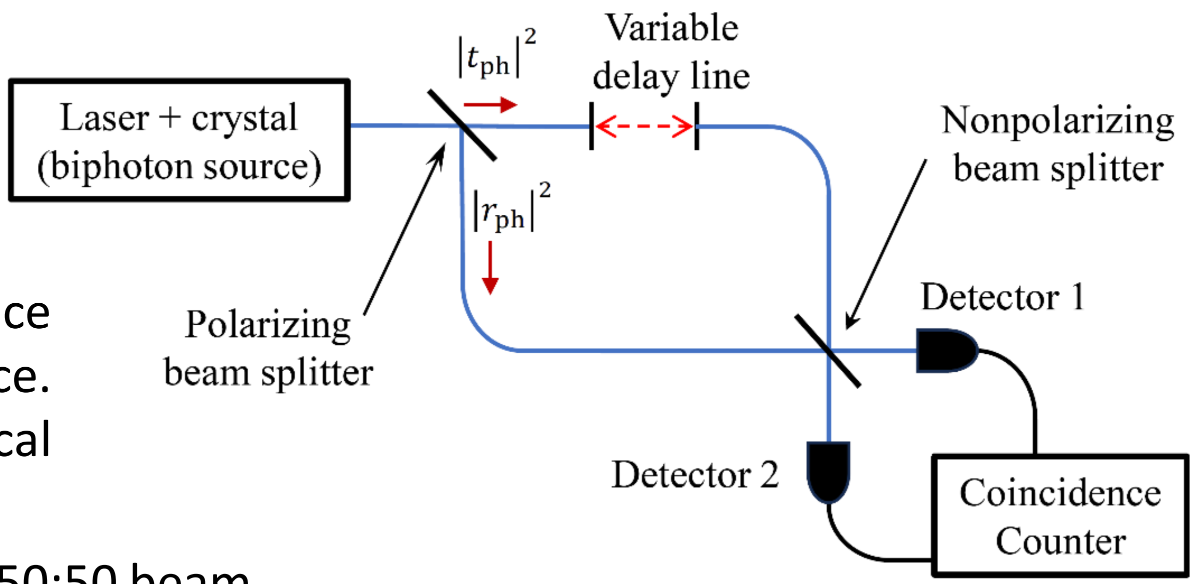
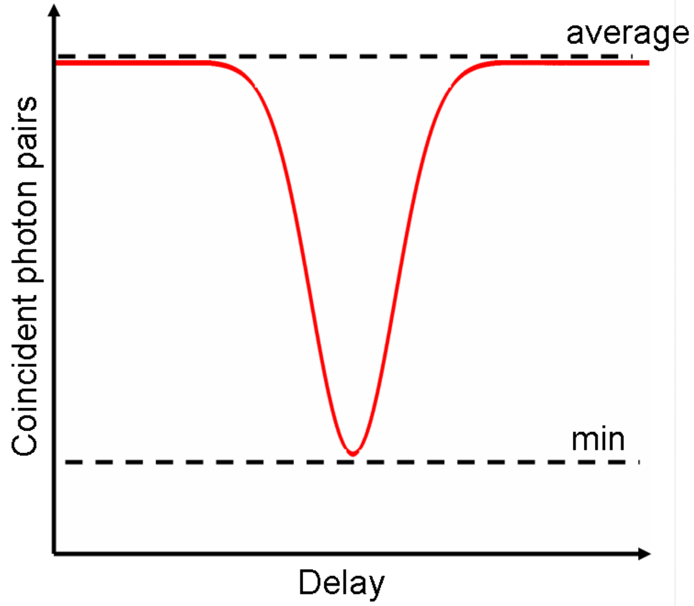
- Distinguishability between two photon particles is achieved if they have different:
  - Polarization
  - Wavelength
  - Arrival time at the *detector*
- *Quantum* interference results in a reduction in coincidence counts below that expected from a classical light source. This reduction is the ***HOM dip***[1] that has no classical analog!
- Precisely one photon leaves each output port of the first 50:50 beam splitter and they are guaranteed to be **identical** and **indistinguishable**.
- They enter the second beam splitter and the indistinguishable photons are detected at the output ports at the same time.
- The variable delay line can change the arrival time at detectors 1 and 2, making the photons distinguishable.
- Indistinguishable photons interfere quantum mechanically at the detector, giving rise to a dip in coincidence counts.





[1] C. K. Hong, Z. Y. Ou, and L. Mandel, *Phys. Rev. Lett.* **59**, 2044 (1987).

Beam splitters and photon number states

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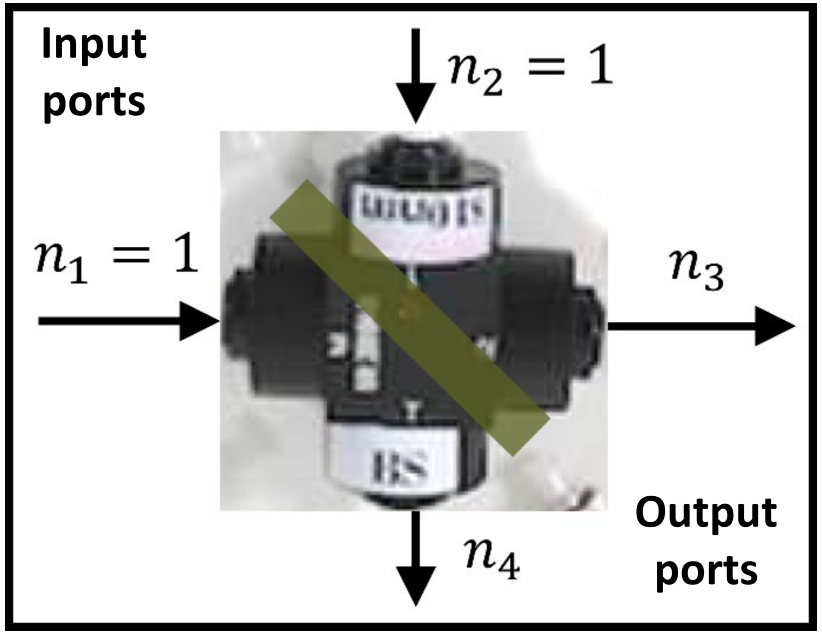
- To quantify how quantum interference is suppressed as the distinguishability of the photons increases, it is convenient to describe interaction with a beam splitter using boson creation ($\hat{b}^\dagger$) and annihilation ($\hat{b}$) operators which act on photon-number states.



Example: A photon is created in input port 1 of the beam splitter: $\hat{b}_1^\dagger|0\rangle_1 = |1\rangle_1$

- Now consider the two-photon **input** port and **output** port *particle number* states:

Input port state: $|n_1, n_2\rangle_{\text{in}} = |n_1\rangle_1 \otimes |n_2\rangle_2 = |n_1\rangle_1|n_2\rangle_2 = |n_1, n_2\rangle_{12}$

Output port state: $|n_3, n_4\rangle_{\text{out}} = |n_3\rangle_3 \otimes |n_4\rangle_4 = |n_3\rangle_3|n_4\rangle_4 = |n_3, n_4\rangle_{34}$

Beam splitters can be described by a *unitary* observable

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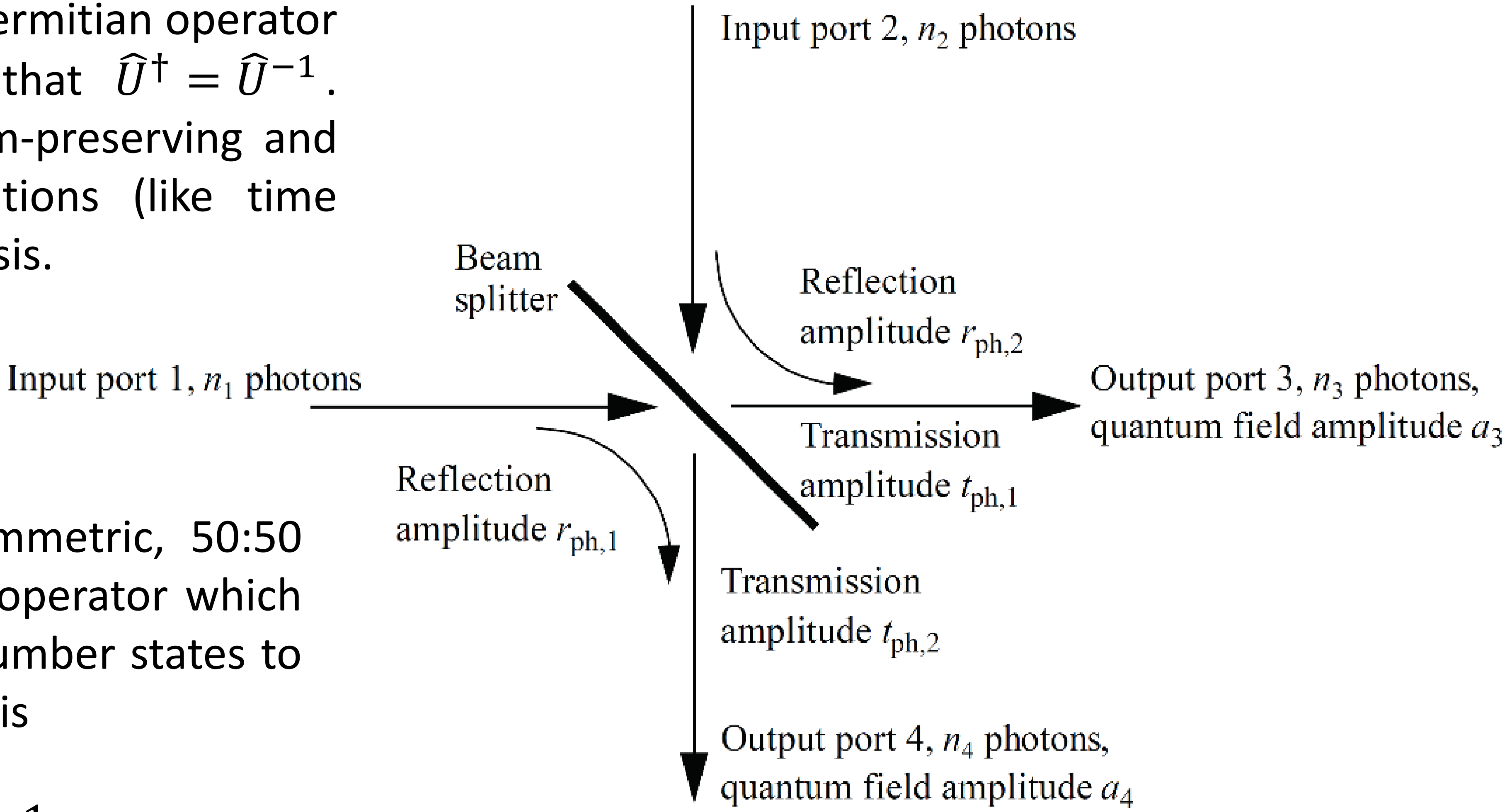
- A unitary observable is a Hermitian operator for which $\hat{U}^{\dagger}\hat{U} = \hat{1}$, so that $\hat{U}^{\dagger} = \hat{U}^{-1}$. These operations are norm-preserving and can represent transformations (like time evolution) or changes in basis.

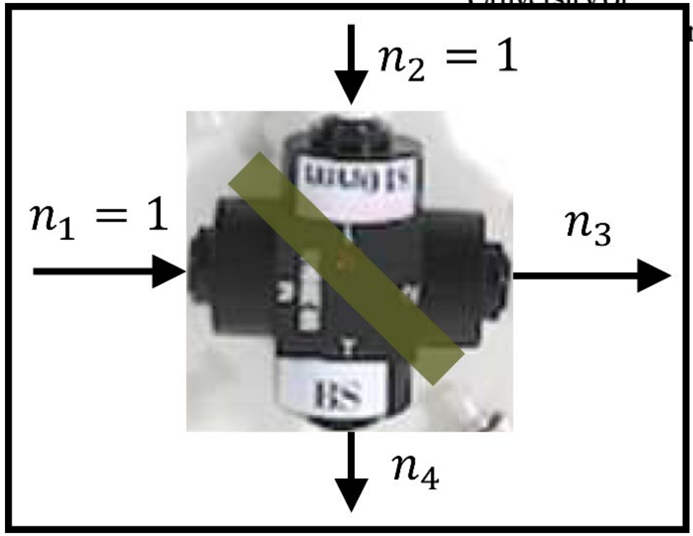


- For an ideal, lossless, symmetric, 50:50 beam splitter, the unitary operator which maps input port photon number states to output port number states is

$$\hat{U}_B = \begin{bmatrix} t_{\mathrm{ph}} & r_{\mathrm{ph}} \\ r_{\mathrm{ph}} & t_{\mathrm{ph}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i} & -1 \\ -1 & \mathrm{i} \end{bmatrix}$$

where the transmission and reflection amplitudes respectively are $t_{\mathrm{ph}} = \mathrm{i}/\sqrt{2}$ and $r_{\mathrm{ph}} = -1/\sqrt{2}$.

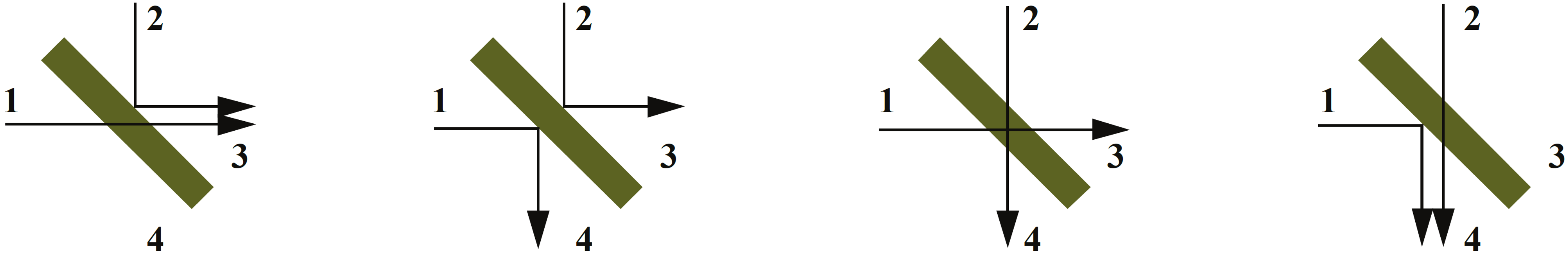
Probability amplitudes of 4 different possible outcomes



- If there is a <u>single identical, indistinguishable photon</u> ***at each*** input port 1 and 2 of the beam splitter, then

$$|1\rangle_1|1\rangle_2 = b_1^\dagger b_2^\dagger |0\rangle_1|0\rangle_2$$

- As illustrated below, there are four different paths the two photons can take to the output ports 3 and 4 and be detected *simultaneously* are



$\frac{\mathrm{i}}{2}(\hat{b}_3^\dagger \hat{b}_3^\dagger)|0,0\rangle_{34}$ $\quad$ $\frac{1}{2}(\hat{b}_3^\dagger \hat{b}_4^\dagger)|0,0\rangle_{34}$ $\quad$ $-\frac{1}{2}(\hat{b}_3^\dagger \hat{b}_4^\dagger)|0,0\rangle_{34}$ $\quad$ $\frac{\mathrm{i}}{2}(\hat{b}_4^\dagger \hat{b}_4^\dagger)|0,0\rangle_{34}$

The probability of detecting two indistinguishable photons at either output port is exactly one-half and the output port at which the two photons are detected is a fundamentally random quantum mechanical process.

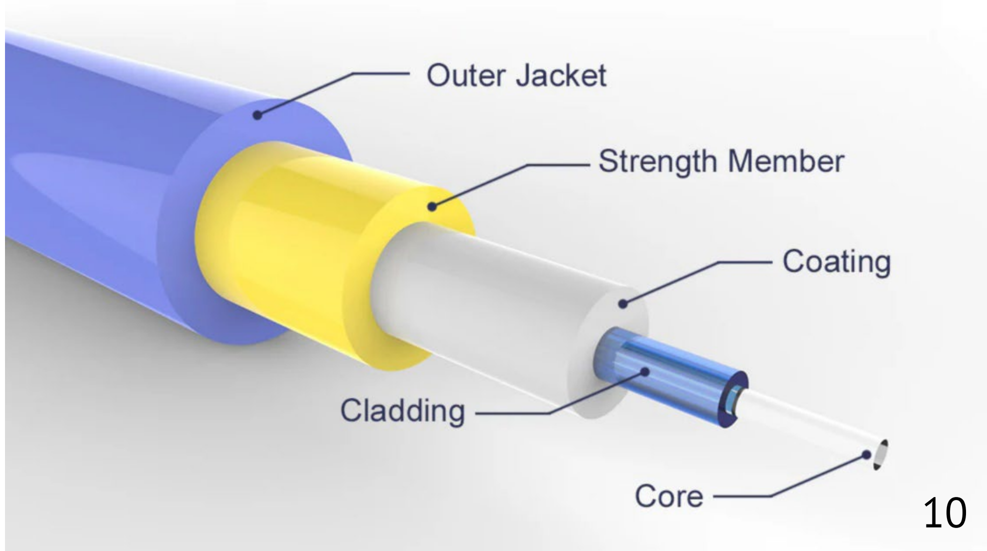
Experiment hardware

Observing the Mandel dip requires an experimental setup that typically includes the following components:

- **Laser source**: This is a stabilized high-power laser diode with single-mode, spectrally narrow line-width, emission peaked at photon energy *E*.

- **Biphoton source**: This a source that emits indistinguishable biphotons via the interaction of laser light with a nonlinear medium. A typical mechanism for biphoton generation is *spontaneous parametric down-conversion* (SPDC) in which a single photon of energy $E(\lambda = 405\text{nm}) = 3.06\text{ eV}$ is converted into two indistinguishable photons with double the wavelength and half the energy $E_{1/2}(\lambda = 810\text{nm}) = 1.53\text{ eV}$.

- **Optical fibers and connectors**: These are used to guide photons between optical elements and to minimize the introduction of noise such as light from other sources. *Polarization-maintaining* fiber optic cables identified by a blue outer jacket are used.



Experiment hardware, continued

- **Beam splitters**: Used to direct photons into different paths or to mix different photon streams coherently. The manipulation of photon states at a beam splitter is modeled as a unitary transformation.

Polarizing beam splitter

Non-polarizing beam splitter

- **Variable delay line**: Used to vary photon path length and control photon distinguishability. Mechanically controlled by a programmable stepper motor.

- **Photon detectors**: Detectors such as avalanche photodiodes (APDs) or superconducting nanowire single-photon detectors (SNSPDs) based on transition edge sensing (TES) are used. These are capable of detecting individual photons with high efficiency and timing resolution.

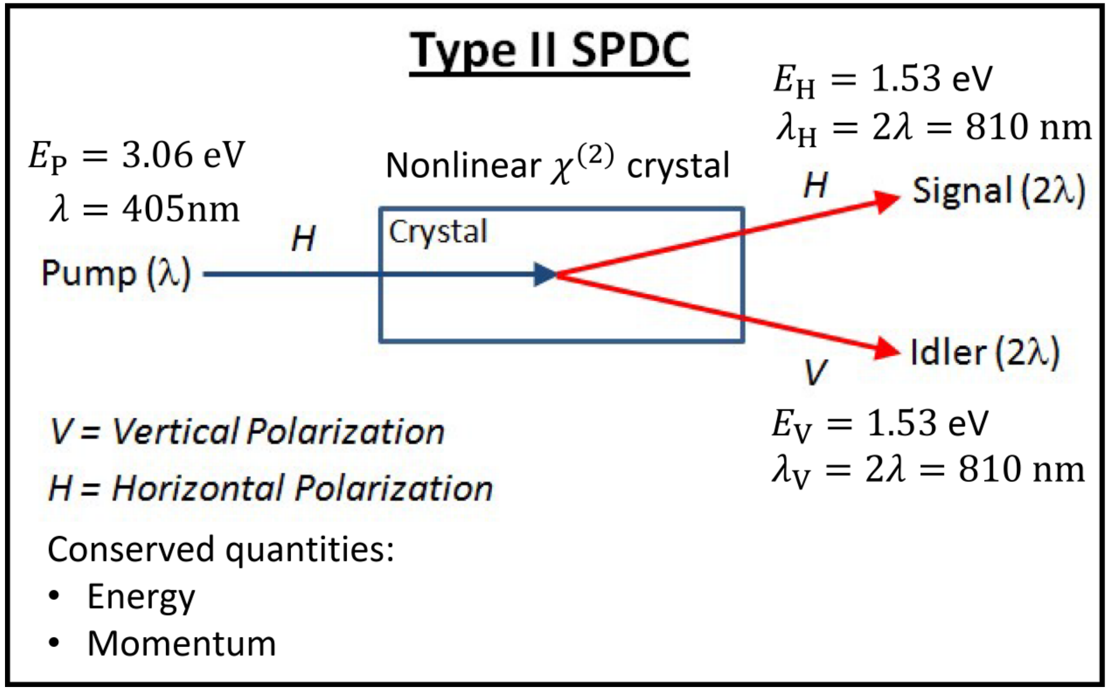
- **Coincidence counter (CC)**: This electronic circuit measures the time interval between electrical pulses that are outputted from the single-photon detectors.

- **Data acquisition system**: To record the counts and analyze the statistics of the detected photons.

SPDC: The process used to generate biphotons

- Photon pairs are created using the *nonlinear* process of spontaneous parametric down-conversion (SPDC).
- Inside the QES2 module, a laser (pump beam) is directed into a nonlinear crystal.
- Two general categories of nonlinear materials for generating biphotons: *waveguides* and ***bulk crystals*** (this is used in our demonstration experiments).
- As photons pass through the crystal, sometimes a photon pair of half the pump's energy is produced; these two photons (the signal and idler photons) have perpendicular polarizations = **Type II SPDC**
- Critical parameters which influence the design of a non-linear crystal:
  - Pump wavelength
  - Pump power
  - Whether pump beam is pulsed or continuous
  - Target spectral and polarization requirements for the signal and idler photons.
- An *inefficient process*: only <u>**1 part in 100 billion**</u> undergoes down conversion to produce identical photon pairs.

QES2 module



PP-KTP: nonlinear crystals used in Type-II SPDC

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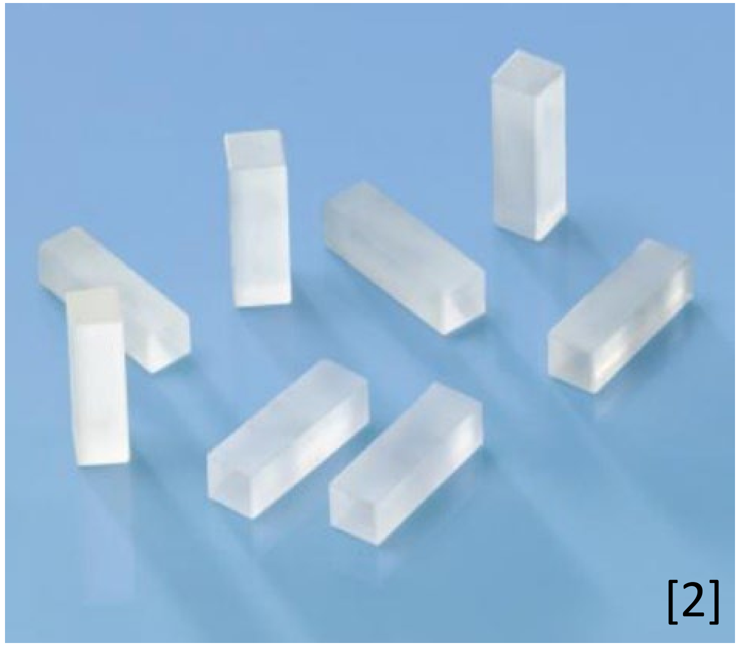
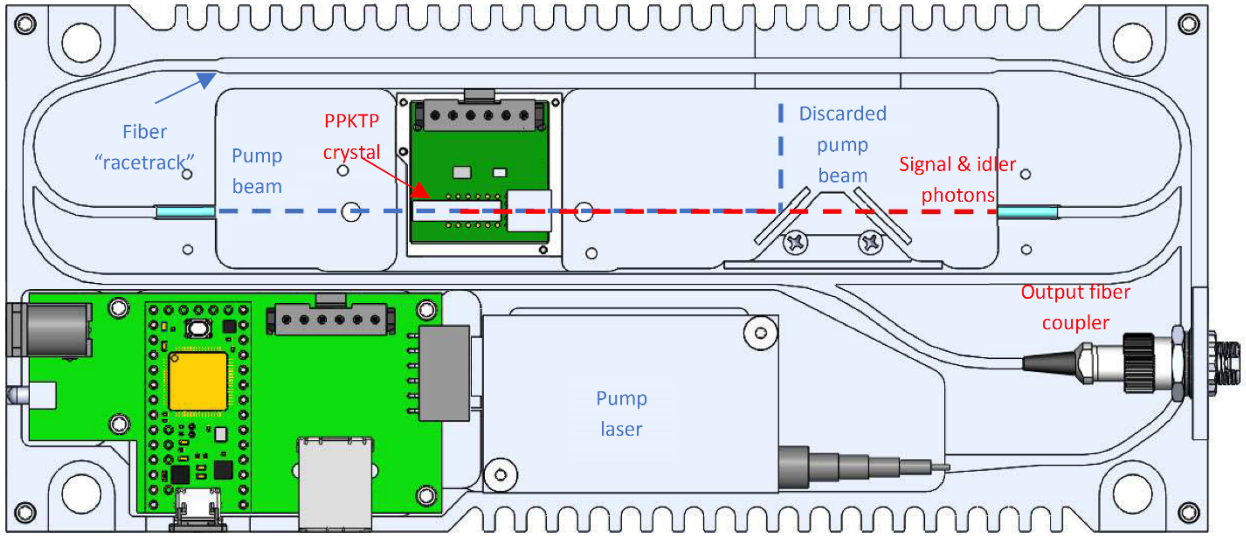
- Optical crystals can exhibit different kinds of optical nonlinearities, which can be used for parametric nonlinear frequency conversion (frequency doublers, optical parametric oscillators) and electro-optic modulators.
- The biphoton module used in this demonstration is a cm-long crystal made out of a **periodically-poled Potassium Titanyl Phosphate** (PP-KTP crystal).
- This crystal down-converts an H-polarized blue (405nm wavelength ~ 1.35fs cycle time ~ 0.244 eV spectral width) pump photon into an H-polarized “red” (810nm) signal photon and a V-polarized “red” (810nm) idler photon.
- The pump light is delivered to the PP-KTP crystal via an optical fiber, and a second fiber carries the signal and idler photons to the output fiber coupler.
- A dichroic mirror before the second fiber separates the two colors of photons and ensures that only the red signal and idler photons get collected by the output optical fiber (thereby eliminating the pump photons).



[2]



[2] KTP crystals (source: EKSMA Optics)

Temperature control of nonlinear crystal in biphoton source

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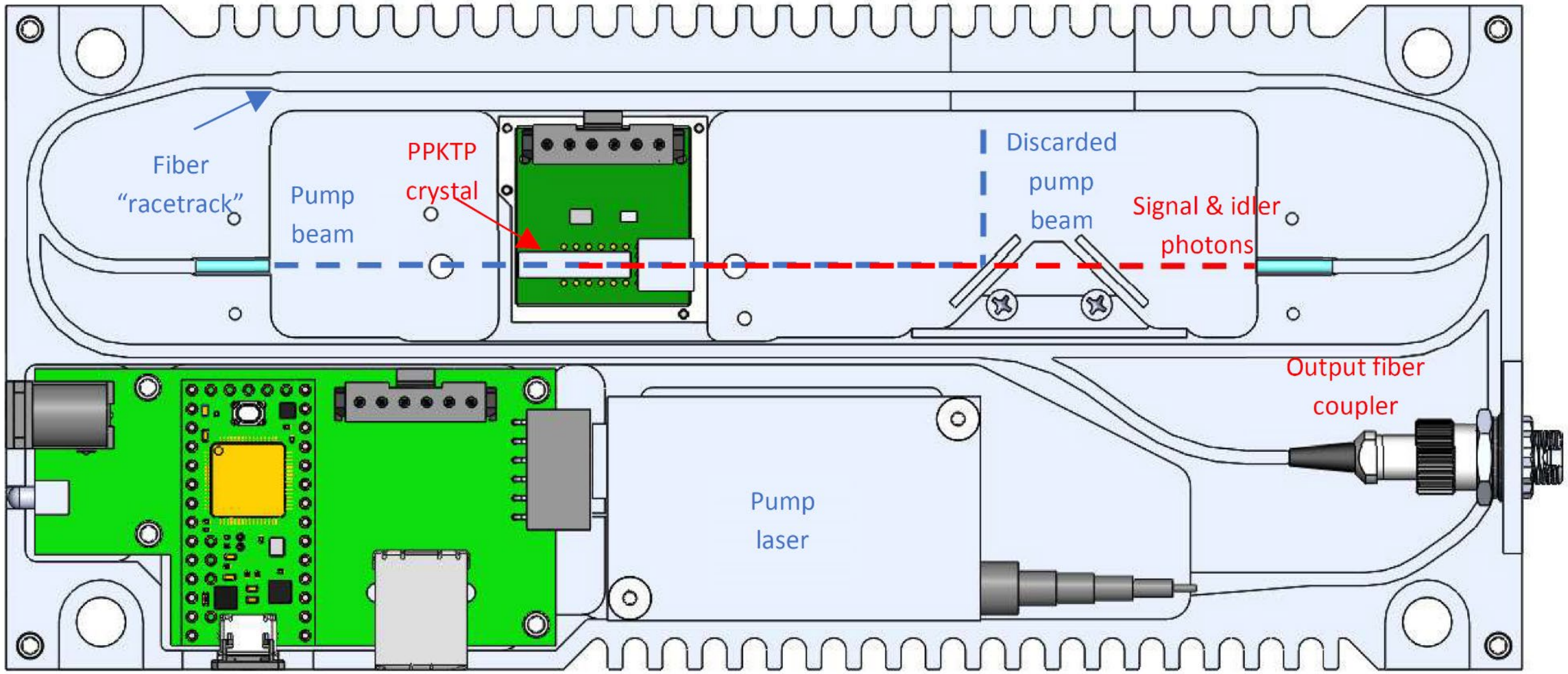
- The QES2 also has a small heater which can change the temperature of the crystal.
- As the temperature of the crystal is changed, the wavelengths of the emitted signal and idler photons will change.
- For a given pump wavelength, there is a unique crystal temperature that will produce signal and idler photons with exactly the same wavelength.
- When the signal and idler photons have the *same* wavelength, the photons are **degenerate**.
- When the signal and idler photons have *different* wavelengths, the photons are **non-degenerate**.



Photon detection: **S**ingle **P**hoton **C**ounting **M**odule (**SPCM**)

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- Measurement = *quantum* state destroyed, converted into *classical* electrical pulse
- When a photon is launched into an SPCM, the device converts the photon into a classical electrical pulse.
- As shown below, a 10ns 2.2V electrical pulse is produced by the SPCM whenever a photon is detected.

- Although such a device provides a convenient method for detecting photons, it has limitations.
- While the 10ns pulse is being produced, the SPCM is unable to produce another pulse – even if additional photons are absorbed.
- Furthermore, after the 10ns electrical pulse is emitted, the SPCM requires an additional 12ns to "reset" before it can measure another photon.
- The total "deadtime" for this SPCM (the amount of time between subsequent photon measurements) is 10ns + 12ns = 22ns.
- Therefore, the SPCM can only measure and resolve a maximum of 37×10^6 photons/second.

Block schematic and HOM experiment parameters

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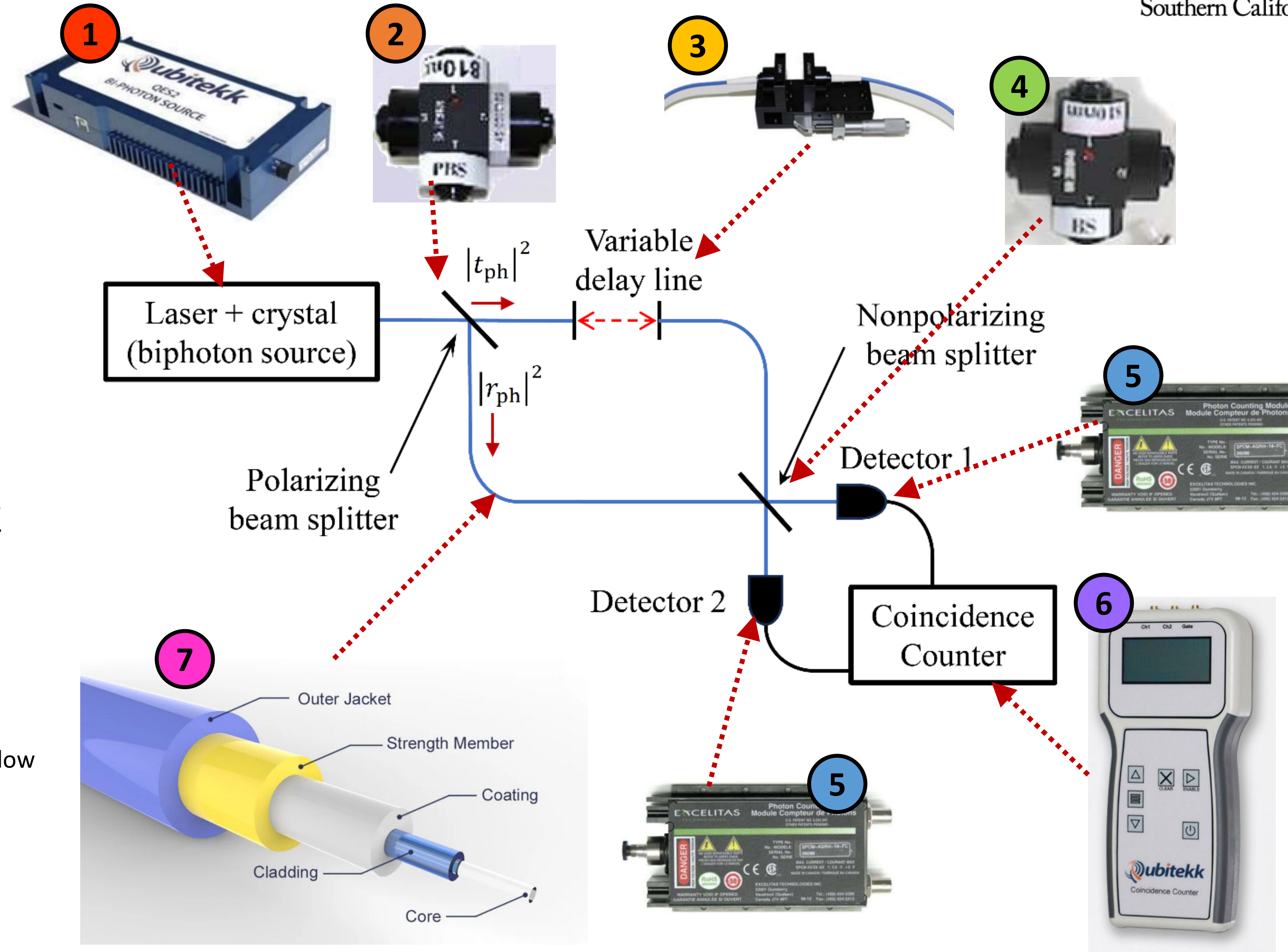
1. Biphoton source
   - tempLaser current, I_{laser}
   - Optical power, $P_{optical}$
   - Crystal erature, $T_{crystal}$
2. Polarizing beam splitter
3. Variable delay line
   - Start position, x_{start}
   - Stop position, x_{stop}
   - Step, Δx
4. Nonpolarizing beam splitter
5. Single photon counting modules (SPCMs)
6. Coincidence counter
   - Coincidence window, t_{window}
   - Dwell time, t_{dwell}
7. Polarization-maintaining fiber optic cables



Experimental demonstration of the Hong-Ou-Mandel effect

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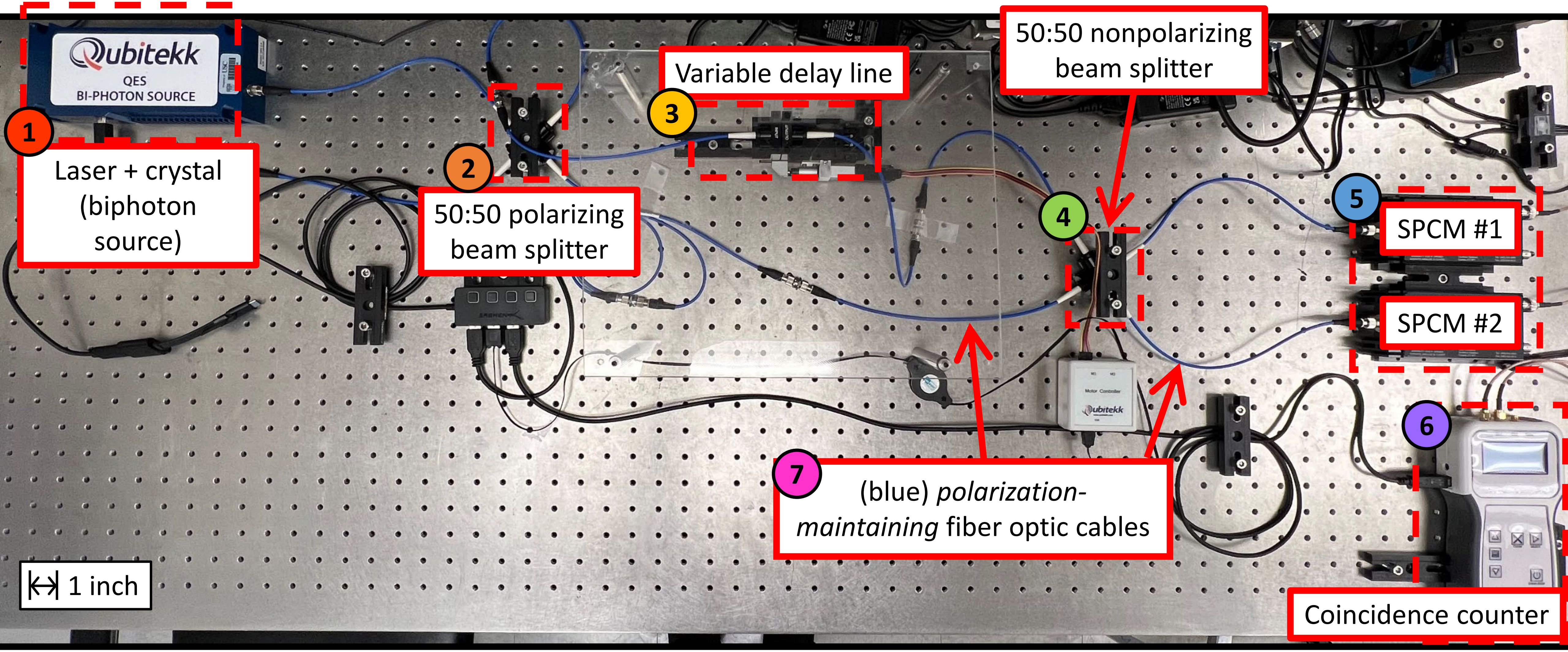
Remote access to HOM experiment



Measurement, and therefore superposition state collapse, ***occurs at the detectors***.

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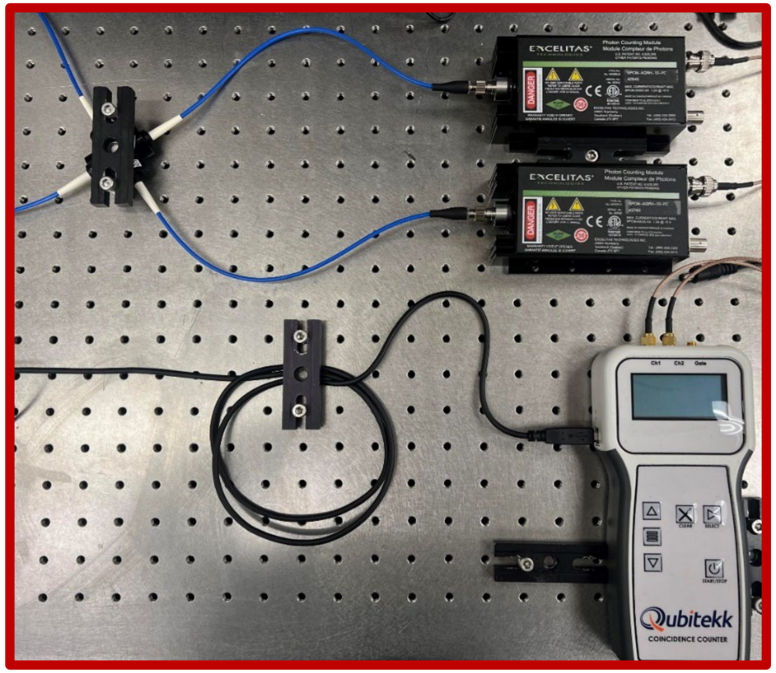

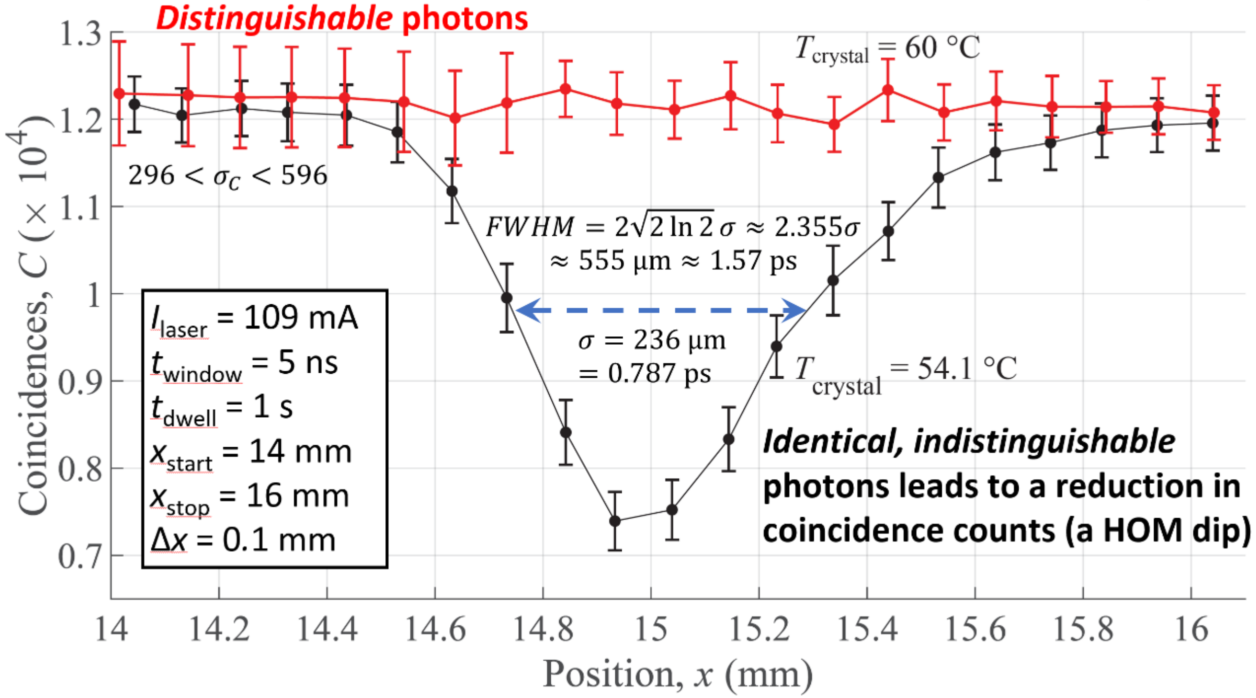

- Symmetry associated with identical, indistinguishable particles results in quantum amplitude interference between different paths through the system.
- The probability of detecting a fixed number of photons at an output port can be dramatically different from classical expectations.
- Two-photon interference is *not* interference of two separate photons at the beam splitter, but rather it is interference of both two-photon amplitudes at the *detectors*.
- Photons don't need to arrive simultaneously at the beam splitter to have their quantum field amplitudes interfere at the detectors; rather it is only that *both* two-photon paths must be indistinguishable.

In summary…

- In quantum mechanics, until the tree is ***observed*** and a ***measurement*** is made, it is in a **linear superposition** of two possible **states**:

- Quantum mechanics is distinguished from classical mechanics by the linear algebra of *non-commuting* **observables**.
- *Unlike* classical mechanics, physical properties of systems (e.g., position, momentum) generally ***cannot*** be **measured** to arbitrary accuracy.
- Quantum mechanical systems exhibit key features that distinguish them from everyday classical objects, including:
    - **Wave-particle duality** (e.g., simultaneous existence of wave (nonlocal) and particle (local) properties)
    - **Linear superposition of particles** (e.g., particle entanglement)
    - **Existence of identical and indistinguishable particles** (no classical analog)
- The HOM dip is a demonstration of all three of these features in a quantum interference experiment.
- Quantum engineering involves exploiting these properties to perform operations that cannot otherwise be accomplished classically. This can result in a **quantum advantage**.